The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme/Continuing Education Programme
Final Examination – 2011/2012
Applied Mathematics – Level 04
AMU 2181/AME 4181 – Mathematical Modeling I

Duration: - Two hours.

Date: - 06-01-2012. **Time: - 9.30 a.m. – 11.30 a.m.**

Answer FOUR questions only.

01. A private hospital consists of 600 beds with complete laboratories, operating rooms and x-ray equipments. In seeking to increase revenues, hospital administration has decided to make a 90 beds addition on a portion of adjacent land currently used for staff parking. The administrators feel that the labs, operating rooms and x-ray equipments are not being fully utilized at present and do not need to be expanded to handle additional patients. The addition of 90 beds, however involves deciding how many beds should be allocated to the medical staff for surgical patients.

The hospital's accounting and medical records departments have provided the following pertinent information. The average hospital stay for a medical patient is 8 days and on the average a medical patient generates 2,28,000 rupees in revenue per stay. The average surgical patient is in the hospital for 5 days and generates 1,51,500 rupees revenue. The laboratory is capable of handling 15,000 tests per year more than it was handling. On the average a medical patient requires 3.1 lab tests and a surgical patient takes 2.6 lab tests. Furthermore, the average medical patient uses one x-ray, whereas the average surgical patient requires two x-rays. If the hospital was expanded by 90 beds, the x-ray department could handle up to 7,000 x-rays without significant additional cost. Finally, the administration estimates that up to 2,800 additional operations could be performed in existing operating room facilities. Medical patients, of course, do not require surgery, whereas each surgical patient generally has one surgery performed.

Formulate a mathematical model to this problem and solve it to determine how many medical beds and how many surgical beds should be added to maximize the revenue. Assume that the hospital is open 365 days a year.

02. Consider the following linear program:

Maximize $Z = x_1 + 2x_2$,

Subject to $x_1 + x_2 \leq 6$,

$x_2 \leq 3$,

$x_1, x_2 \geq 0$.

(i) Find all the basic feasible solutions and corresponding reduced costs of the non-basic variables.

(ii) Identify the optimal basic feasible solution and find the optimal objective value.

03. A factory works 8 hours a day, producing three products viz. A, B and C. Each of these products is processed in three different operations viz. 1, 2 and 3. The processing times in minutes for each of these products in each of the operations are given in below table along with utilization of the processes and the cost and price in rupees for each of these three products which have unlimited demand:

	Processing time (rains.) in operation			Cost/Unit	Price/Unit
product	1	2	3	(Rs.)	(Rs.)
A	4	3	1	10	16
B	2	1	4	8	12
C	3	4	5	5	10
Utilization	80%	70%	90%		

(i) Determine the optimal product mix using the simplex method.

(ii) Give the interpretations for the values obtained in the final simplex table.

04. Consider the following linear program problem:

Minimize $Z = 2x_1 + 15x_2 + 5x_3 + 6x_4$,

Subject to $x_1 + 6x_2 + 3x_3 + x_4 \geq 2$,

$-2x_2 + 5x_2 - x_3 + 3x_4 \leq -3$,

$x_1, x_2, x_3, x_4 \geq 0$.

(i) Write the dual of this problem.

(ii) Solve the dual problem using simplex method and hence, write the solution to the primal problem.

05. Use Big-M method to solve each of the following linear program problem:

(a) Maximize $Z = 3x_1 - x_2$,

Subject to $2x_1 + x_2 \geq 2$,

$x_1 + 3x_2 \leq 3$,

$x_2 \leq 4$,

$x_1, x_2 \geq 0$.

(b) Minimize $Z = 3x_1 + 5x_2$,

Subject to $x_1 + 4x_2 \geq 60$,

$-2x_1 - x_2 \leq -50$,

$x_1, x_2 \geq 0$.

06. Hi Dec produces two models of electronic gadgets using the resources resistors, capacitors and chips. The following table summarizes the data:

Unit Resources			
Resource	Model 1 (units)	Model 2 (units)	Maximum Availability (units)
resistors	2	3	1200
capacitors	2	1	1000
chips	0	4	800
Unit Profit(Rs)	3	4	

Let X_1 and X_2 be the units produced of Model 1 and Model 2 respectively.

Following are the linear programming model and its associated optimal tableau:

Maximize $Z = 3x_1 + 4x_2$,

Subject to $2x_1 + 3x_2 \leq 1200$ (Resistors),

$2x_1 + x_2 \leq 1000$ (Capacitors),

$4x_2 \leq 800$ (Chips),

$x_1, x_2 \geq 0$.

Optimal Simplex Tableau

Basic	X_1	X_2	S_1	S_2	S_3	Solution
Z	0	0	5/4	1/4	0	1750
X_1	1	0	-1/4	3/4	0	450
S_3	0	0	-2	2	1	400
X_2	0	1	1/2	-1/2	0	100

(a) Determine the status of each resource.

(b) In terms of the optimal profit, determine the worth of a resistor, a capacitor and a chip.

(c) Determine the range of applicability of the dual prices for each resource.

(d) If the available number of resisters is increased to 1300 units, find new optimum solution.

(e) If the available number of chips is reduced to 350 units, will you be able to determine the new optimum solution directly from the given information? Explain.

(f) If the availability of capacitors is limited by the range of applicability computed in (c), determine the corresponding range of the optimal profit and the corresponding range for the number to be produced of models 1 and 2.

(g) A new contractor is offering to sell Hi Dec additional resistors at 40 cents each but, only if Hi Dec would purchase at least 500 units. Determine whether Hi Dec should accept the offer or not?