The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2011/2012
APU 3143-Mathematical Methods
APPLIED MATHEMATICS-LEVEL 05

Duration: Two Hours.

Date: 20.01.2012	**Time: 1.30 p.m.- 3.30p.m.**

Answer FOUR questions only.

1. (i) Show that $L^{-1}\left\{\dfrac{s}{(s-2)^2(s+1)}\right\}=e^{2t}\left(\dfrac{t^4}{36}+\dfrac{t^3}{54}-\dfrac{t^2}{54}+\dfrac{t}{81}-\dfrac{1}{243}\right)+\dfrac{e^{-t}}{243}$,

   (ii) Show that

$$L^{-1}\left\{\frac{e^{-4s}-e^{-7s}}{s^2}\right\}=\begin{cases}0 & 0<t<4\\ t-4 & 4<t<7\\ 3 & t>7\end{cases};$$ where L represents the Laplace operator.

   (iii) Use the convolution theorem to find the inverse Laplace transform of each of the following:

   (a) $\dfrac{1}{(s^2+4)(s+1)^2}$ (b) $\dfrac{1}{s^2(s^2-a^2)}$.

2. Solve each of the following boundary value problems using the Laplace transform method.

   (i) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+y=2t$, subject to $y(0)=0,\ y'(0)=0$.

   (ii) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+101y=5\sin 10t$, subject to $y(0)=0,\ y'(0)=20$.

   (iii) $2\dfrac{d^2y}{dt^2}+50y=100\sin\omega t$, subject to $y(0)=0,\ y'(0)=0$.

   (iv) $\dfrac{d^4y}{dt^4}=E$, subject to $y(0)=y''(0)=y(L)=y''(L)=0$; where E is a constant.

3. Find the characteristic values and characteristic functions of the Sturm-Liouville problem

   given by $\dfrac{d^2y}{dx^2}+\lambda y=0,\qquad y(0)+y'(0)=0,\qquad y(\pi)-y'(\pi)=0.$

4. (i) Define an orthonormal system.

(ii) Obtain the formal expansion of the function f where $f(x) = \pi x - x^2$, $0 < x < \pi$ as a series of orthonormal characteristic functions $\{\phi_n\}$ of the Sturm-Liouville problem given below.

$$\frac{d^2y}{dx^2} + \lambda y = 0$$
$$y(0) = 0, \quad y(\pi) = 0$$

Discuss the convergence of the above formal expansion.

5. Let $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$, $-L \le x \le L$ be the Fourier series of f.

Determine the coefficients a_0, a_n and b_n for $n = 1, 2, 3, \ldots$.

Show that the Fourier series of the function

$f(x) = \cos \mu x$ (μ is not an integer), $-\pi \le x \le \pi$

is given by

$$\frac{2\mu \sin \mu\pi}{\pi}\left[\frac{1}{\mu^2} + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos nx}{(n^2 - \mu^2)}\right].$$

6.(i) Let $f(x)$ be a function defined in the interval $0 < x < \pi$. The Fourier cosine series of $f(x)$ is given by

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx, \text{ where } a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx \text{ and } a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

Show that $\frac{4}{\pi}\int_0^{\pi}[f(x)]^2\,dx = \pi a_0^2 + 2\pi\sum_{n=1}^{\infty} a_n^2$.

(ii) A function $f(x)$ is defined over the range

$$0 < x < 4 \text{ as } f(x) = \begin{cases} x, & 0 < x \le 2 \\ 4 - x & 2 < x < 4. \end{cases}$$

(a) Find the cosine and sine expansions of $f(x)$.

(b) Sketch the graph for each expansion.