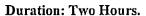
The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2011/2012

AMU3187/ AME 5187- Mathematical Methods II

APPLIED MATHEMATICS-LEVEL 05



Date: 20.01.2012

Time: 1.30 p.m.- 3.30p.m.

Answer FOUR questions only.

1. Consider the periodic function f(x) defined by

$$f(x) = \begin{cases} -\cos x & \text{for } -\pi \le x < 0\\ \cos x & \text{for } 0 < x \le \pi \end{cases}$$

and
$$f(x+2\pi) = f(x)$$
.

- (i) Sketch the graph of f(x) for two periods.
- (ii) Find the Fourier series of f(x).

(iii) Using part (ii) show that
$$\frac{\pi\sqrt{2}}{16} = \frac{1}{1.3} - \frac{3}{5.7} + \frac{5}{9.11} - \cdots$$

2. Consider the boundary value problem:

$$\frac{d^2y}{dx^2} + \mu y = 0$$

y'(0) = 0, y'(1) = 0

- (i) Show that this is a Sturm-Liouville problem.
- (ii) Find the eigenvalues and eigenfunctions of the problem.
- (iii) Verify that the eigenfunctions are mutually orthogonal in the interval $0 \le x \le 1$.
- (iv) Obtain a corresponding set of orthonormal functions in the interval $0 \le x \le 1$.
- 3. Consider the function f(x) defined by

$$f(x) = 2x, \quad 0 \le x \le \pi$$

- (i) Find the Fourier sine series and cosine series of f(x) on $0 \le x \le \pi$.
- (ii) Sketch the graphs of f(x) for the two series.

(4) Let $J_n(x)$ be the Bessel function of order n given by the expansion

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n.$$

- (i) Show that $J_n(x)$ is an even function when n is even and an odd function when n is odd.
- (ii) Verify each of the following identities for n=1, 2, 3, ...

(a)
$$J_{-n}(x) = (-1)^n J_n(x)$$

(b)
$$\frac{d}{dx} \left\{ x^n J_n \left(x \right) \right\} = x^n J_{n-1} \left(x \right).$$

(c)
$$\frac{d}{dx} \{x^{-n} J_n(x)\} = -x^{-n} J_{n+1}(x).$$

5. The Rodrigues' formula for the n^{th} degree Legendre polynomial is given by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$
, $n=0, 1, 2, ...$

Using it prove the following identities:

(i)
$$P'_{n+1}(x) = (n+1)P_n(x) + xP'_n(x)$$

(ii)
$$(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$$

(iii)
$$(x^2-1)P'_n(x) = nxP_n(x) - nP_{n-1}(x)$$
.

(iv)
$$P_n(-x) = (-1)^n P_n(x)$$
 and $P'_n(-x) = (-1)^{n+1} P'_n(x)$

(6) Solve the following boundary value problem.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b.$$

$$u(x,0) = f_1(x), \quad 0 < x < a$$

$$u(x,b) = f_2(x), \qquad 0 < x < a$$

$$u(0, y) = g_1(y), \quad 0 < y < b$$

$$u(a, y) = g_2(y), \quad 0 < y < b$$