The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination - 2011/2012
Applied Mathematics – Level 05
AMU3183/AME5183- Numerical Methods II

Duration: Two hours

Date : 23.12.2011 **Time: 1.30p.m. - 3.30p.m.**

Answer Four Questions Only.

1.

a) With the usual notations, derive the Trapezoidal and Simpson's rules.

b) A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation,

$$t=\int_{v(t_0)}^{v(t)}\frac{m}{R(u)}du.$$

Suppose that $R(v)=-v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in meters/seconds. If $m=10\,kg$ and $v(0)=10\,ms^{-1}$, approximate the time required for the particle to slow to $v=5\,ms^{-1}$.

I) Use the composite Trapezoidal rule with $h=0.5$
II) Use the composite Simpson's rule with $h=0.5$
III) Compare these approximations to the actual value.

2.

a) Write down the general form of the Taylor series method.

b) Obtain Euler method using Taylor series method.

c) Find the solutions $u(0.1)$ and $u(0.2)$, of the initial value problem $u'=x(1-2u^2)$; $u(0)=1$ using the first three non zero terms of the Taylor series method with $h=0.1$.

3.

a) Explain the 4th order Runge-Kutta method for solving ordinary differential equations.

b) The initial value problem $y' = \sqrt{x+y}$, $y(0) = 1$ is given. Find the value of $y(0.2)$ with $h = 0.1$ by using Runge-Kutta method of 4th order.

c) Use the Runge-Kutta method of 2nd order to find $y(1.1)$ with $h = 0.1$ for the initial value problem, $y' = 3x + y^2$; $y(1) = 1.2$.

4.

An engineer involved in the design of automobiles uses an experimental system for studying the motion of a wide variety of vehicular devices in a full-scale laboratory environment. One particular test involves an accurate measurement of the displacement x of the vehicle as a function of time t. This information is then used to determine the velocity v, the acceleration A, and the rate of the change of acceleration F as functions of time. In a given experiment, the displacement x was measured over a time range of $0s$ to $10s$, at steps $0.1s$. Some of the results obtained are as follows;

$t(s)$	0.0	0.1	0.2	0.3	0.4	0.5	0.6
$x(t)$	0.0	0.8733	1.8224	2.8611	4.0032	5.2625	6.6528

From these data, compute v, A and F at $t = 0s$, employing forward differences, with the step size Δt of $0.1s$.

5.

a) Consider a 3rd order Lagrange polynomial and show that it may be recast in the general form of a 3rd order polynomial given by $a_0 + a_1x + a_2x^2 + a_3x^3$. Obtain the relationship between the coefficients of the two polynomials.

b) If the model $y = ae^{bx}$ is fitted by method of least squares for the following data, determine the constants a and b.

x	0.0	0.5	1.0	1.5	2.0	2.5
y	0.10	0.45	2.15	9.15	40.35	180.75

Hence find y when $x = 3$.

6.

a) Using Euler's method obtain an approximation to the solution of the initial value problem

$$\frac{dy}{dx} = y(e^x - 1);\ y(0) = 1;\ \text{at } x = 0.3.\ \text{Take the step size } h = 0.1.$$

b) Show that the exact solution of the above initial value problem is given by,
$\ln y = e^x - x - 1$.
Compare your approximation with the exact value.

c) The intensity (I) of a certain colour of a wall painting is deteriorating at a rate proportional to the time (t) and the existing intensity of the colour. Furthermore, deteriorating rate spends on periodic weather pattern, which is described by the function $\sin\left(\frac{\pi}{2}(t-1)\right)$.

The relevant differential equation to this phenomenon is given by

$$\frac{dI}{dt} = -tI + \sin\left(\frac{\pi}{2}(t-1)\right).$$

Initially (i.e. when $t = 0$), I is considered as 1.

Using the Euler method by taking $h = 0.1$ as the step size, generate an iterative formula to approximate the values of I. Hence approximate the colour intensities at $t = 0.1$.