The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination - 2011/2012

Applied Mathematics – Level 05

AMU3182/AME5182- Mathematical Methods I

Duration: Two hours

Date : 04-01-2012 **Time: 9.30a.m. - 11.30a.m.**

Answer Four Questions Only.

01. (a) Find the general solution of each of the systems of simultaneous differential equations given below:

(i) $\frac{dx_1}{dt} = 3x_1 + 3x_2 + e^{-t}$

$\frac{dx_2}{dt} = -x_1 - x_2 + e^{2t}$

(ii) $\dot{x}_1 = 2x_1 + x_2$

$\dot{x}_2 = -4x_1 + 2x_2$

(b) Find the sinusoidal solution of the system of the equations

$\ddot{x}_1 + 3\dot{x}_2 + 2x_1 = \sin 3t$

$\ddot{x}_2 + \dot{x}_1 = \cos 3t$

02. (a) Find the general solution of the simultaneous differential equations,

$\frac{\partial u}{\partial x} = ye^{xy}$

$\frac{\partial u}{\partial y} = xe^{xy} + 2y$

(b) Find the equations of the characteristic curves for the partial differential equation

$\frac{\partial^2 u}{\partial x^2} - y\frac{\partial^2 u}{\partial x \partial y} = 0 \qquad (y > 0)$ where $u = u(x, y)$.

(c) Using the integrating factor method, find the general solution of the partial differential equation

$\frac{\partial u}{\partial x} + u \cot x = \sin x$, where $u(x, y)$.

03. (a) Solve the following boundary value problem

$$u''(x)-4u'(x)+5u(x)=0,\quad u(0)=0,\quad u'\left(\frac{\pi}{2}\right)-2u\left(\frac{\pi}{2}\right)=1.$$

(b) Find all the eigen values and eigen functions of $u''(x)+\lambda u(x)=0$, given that $u(0)=u'(l)=0$.

04. Obtain the general solution for $4u_{xx}+5u_{xy}+u_{yy}+u_x+u_y=2$ by reducing it to the standard form $u_{\xi\eta}=\frac{1}{3}u_\eta-\frac{8}{9}$, where $\xi=y-x,\ \eta=y-\frac{x}{4}$.

05. (a) Using the change of variable $x=\tan t,\ \left(-\pi/2<t<\pi/2\right)$ solve the homogeneous differential equation,

$$\left(1+x^2\right)^2\frac{d^2y}{dx^2}+\left(1+x^2\right)(2x+1)\frac{dy}{dx}-6y=0.$$

(b) Solve the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=4x-20\cos 2x.$$

06. Use the method of separation of variables to solve the partial differential equation

$$\frac{\partial^2u}{\partial t^2}=c^2\frac{\partial^2u}{\partial x^2},\ 0<x<l$$

where c is a constant, $u(0,t)=0=u(l,t),\ t>0$ and

$u(x,0)=f(x),\ u_t(x,0)=h(x),\ 0<x<l.$