THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
APU3146/APE5146 - Operations Research
FINAL EXAMINATION 2012/2013
Duration: Two hours

Date: 13.12.2013	**Time: 09.30 a.m- 11.30 a.m**

Answer four questions only.

(1) (i) Explain minimax and maximin principles used in the theory of games.

(ii) Let A = $\lfloor a_{ij} \rfloor$ be the payoff matrix for a two-person zero sum game. If $\bar{v}$ denotes the maximin value, $\underline{v}$ be the minimax value of the game, then Show that $\bar{v} \geq \underline{v}$.

(iii) There are two players in a game, say player A and player B. Player A has Rs.2 coin and Rs. 5 coin, and player B has Rs.1 coin and Rs.10 coin. Each player selects a coin from the other player without knowing what coin the other player selected. If the total of the coins selected is odd, player A gets both of the two coins that were selected, but if the total is even, player B gets both coins.

a) Construct the payoff matrix with respect to the player A.
b) Determine the optimal strategies for player A and player B.
c) Find the value of the game.

(2) (i) Briefly explain the following terms:

a) Queue discipline
b) Service mechanism
c) Service channel

(ii) Patients arrive at the Government hospital for emergency service at the rate of one every hour. Currently, only one emergency case can be handled at a time. Patients spend on average of 20 minutes receiving emergency care.

a) What is the probability that a patient arriving at the hospital will have to wait?

b) Find the average length of the queue that forms.

c) Find the average time a patient spends in the system.

d) What is the probability that there will be five or more patients waiting for the service?

e) Determine the fraction of the time that there are no patients.

f) Find the average service time need to be decreased to keep the average time in the system less than 25 minutes.

(3) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 km down the river. Tankers arrive according to a Poisson distribution with a mean of 1 for every 2 hours. It takes for an unloading crew, on the average, 10 hours to unload a tanker, the unloading time follows an exponential distribution.

(i) How many tankers are at the port on the average?

(ii) How long does a tanker spend at the port on the average?

(iii) What is the average arrival rate at the overflow facility?

(4) A library wants to improve its service facilities in terms of the waiting time of its borrowers. The library has two counters at present and borrowers arrive according to Poisson distribution with arrival rate 1 every 6 minutes and service time follows exponential distribution with a mean of 10 minutes. The library has relaxed its membership rules and a substantial increase in the number of borrowers is expected. Find the number of additional counters to be provided if the arrival rate is expected to be twice the present value and the average waiting time of the borrowers must be limited to half the present value.

(5) (i) Define the term inventory.

(ii) What are the advantages and disadvantages of having inventories?

(iii) Formulate the Economic Order Quantity (EOQ) model in which demand is not uniform and production rate is infinite.

Let $t_1, t_2, \ldots, t_n$ denote the times of successive production runs, such that

$t_1 + t_2 + \ldots\ldots + t_n = 1$ year

(iv) A company uses annually 24000 units of a raw material which costs Rs. 1.25 per unit. Placing each order costs Rs. 22.50, and the carrying cost is 5.4 per cent per year of the average inventory. Find the Economic Order Quantity and the total inventory cost.

(5) (i) Briefly explain the following terms in inventory management.

a) Carrying cost

b) Shortage cost

c) Ordering cost

(ii) Derive EOQ model for deterministic demand when replenishment rate is infinite and shortages are permitted.

(iii) A particular item has a demand of 9000 units per year. The cost of one procurement is Rs.100 and the holding cost per unit is Rs. 2.40per year. The replacement is instantaneous and the cost of shortage is Rs. 5 per unit per year. Determine

a) the lot size,

b) the number of orders per year,

c) the time between orders and

d) the total cost per year if the cost of one unit is Rs.1.

Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$$P(\text{queue size} \geq \text{n}) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda}$$

$$E(m) \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$V(n) = \frac{\lambda\mu}{(\mu - \lambda)^2}$$

$$E(w) = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$E(v) = \frac{1}{\mu - \lambda}$$

(M/M/1): (N/FIFO) Queueing System

$$E(n) = \frac{\rho\left[1 - (N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(m) = \frac{\rho^2\left[1 - N\rho^{N-1} + (N-1)\rho^N\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ;1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ;n \geq C \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C \frac{C\mu}{C\mu - \lambda} \right]^{-1}$$

$$E(m) = \frac{\lambda\mu \left(\frac{\lambda}{\mu} \right)^C P_o}{(C-1)!(C\mu - \lambda)^2}$$

$$E(n) = E(m) + \frac{\lambda}{\mu}$$

$$E(w) = \frac{1}{\lambda} E(m)$$

$$E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; 0 \le n \le C \\ \frac{1}{C^{n-1} C!} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; C \le n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C \left\{ 1 - \left(\frac{\lambda}{C\mu} \right)^{N-C+1} \right\} \frac{C\mu}{C\mu - 1} \right]^{-1} & ; \frac{\lambda}{C\mu} \ne 1 \\ \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C (N - C + 1) \right]^{-1} & ; \frac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2} \left[1 - \rho^{N-C+1} - (1-\rho)(N - C + 1)\rho^{N-C} \right]$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!}$$

$$E(v) = {[E(n)]} / {\lambda'}, \quad \text{where } \lambda' = \lambda(1 - P_N)$$

$$E(w) = E(v) - \frac{1}{\mu}$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \ 0 \le n < R \\ \binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right]$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n$$

$$E(v) = \frac{E(n)}{\lambda\left[K - E(n)\right]}$$

$$E(w) = \frac{E(m)}{\lambda\left[K - E(n)\right]}$$