THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
AMU3186/AME5186- Quantum Mechanics
FINAL EXAMINATION- 2012/2013
Duration: Two Hours.

Date: 19.12.2013	**Time: 1.30 p.m. to 3.30 p.m.**

Answer **FOUR** Questions only.

1.(i) Show that for Compton scattering

$$\delta\lambda = \lambda^1 - \lambda = \lambda_c(1-\cos\phi)$$

where $\lambda_c = \dfrac{h}{mc}$ is called Compton wave length, λ is wave length of the incident X-ray and λ^1 is the wave length of X-ray, scattered through an angle ϕ.

(ii) Calculate the Compton shift.

(iii) Show that when $\lambda^1 >>> \lambda_c$, the ratio T of the electron kinetic energy after the collision, to the initial energy is given approximately by,

$$\frac{T}{h\nu} = \frac{\lambda_c}{\lambda}(1-\cos\varphi).$$

you may take $m = 9.108\times10^{-31}Kg$, $c = 3\times10^8 ms^{-1}$, $h = 6.625\times10^{-34}Js$ and $\phi = 30^0$

2.(i) Let $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ be given operators. Prove the following commutator relations

(a) $\left[\hat{A}, \hat{B}\hat{C}\right] = \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right]$

(b) $\left[\hat{A}^2, \hat{B}\right] = \hat{A}\left[\hat{A}, \hat{B}\right] + \left[\hat{A}, \hat{B}\right]\hat{A}$

(c) $\left[\hat{A}+\hat{B}, \hat{C}+\hat{D}\right] = \left[\hat{A}, \hat{C}\right] + \left[\hat{A}, \hat{D}\right] + \left[\hat{B}, \hat{C}\right] + \left[\hat{B}, \hat{D}\right]$

(d) $\left[\hat{A}, \left[\hat{B}, \hat{C}\right]\right] + \left[\hat{B}, \left[\hat{C}, \hat{A}\right]\right] + \left[\hat{C}, \left[\hat{A}, \hat{B}\right]\right] = 0$

(ii) If $\hat{A} = \frac{d}{dx}$ and $\hat{B} = \hat{x}$, check whether $\hat{A}$ and $\hat{B}$ commute.

(iii)Show that $\hat{T} = -\frac{\hbar^2}{2m}\nabla^2$, where T is the kinetic energy of a particle and ∇ has a standard meaning.

3. The parity operator $\hat{\Pi}$ is defined by the operation $\hat{\Pi}\Psi(x) = \Psi(-x)$.Show that

(i) Show that it is a linear operator.

(ii) Show that it is a Hermitian operator.

(iii) Find out the eigen values of this operator Π.

(iv) If the potential energy V is an even function, then show that $\hat{\Pi}$ commutes with the Hamiltanian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$.

4. Consider the potential step

$$V(x) = \begin{cases} 0 & ; \; x < 0 \\ V_0 > 0 & ; \; x > 0 \end{cases}$$

A particle of mass m and energy E moves in the positive x direction and meets the above potential step.

Show that the Schrodinger equation for the case $E > V_0$ has solution

$$u(x) = \begin{cases} Ae^{ik_1x} + Be^{-ik_1x} & ;x < 0 \\ Ce^{ik_2x} & ;x > 0 \end{cases}$$

where $k_1^2 = \frac{2mE}{\hbar^2}$ and $k_2^2 = \frac{2m(E - V_0)}{\hbar^2}$.

Also,

(i) Find the relationship between A, B and C.

(ii) Obtain the reflection (R) and transmission (T) coefficients.

(iii) Show that R+T=1.

5. Consider a particle with normalized function Ψ given by

$$\Psi(x,t)=\begin{cases} A\cos\left(\dfrac{\pi x}{2a}\right).e^{-i\beta t} & ;0\le x\le a \\ 0 & ;\text{elsewhere} \end{cases}$$

where a and β are positive constants.

(a) Determine the normalization constant A.

(b) Fine the expectation values of

(i) the position (ii) the square of the position

(iii) the momentum of the particle

(c) Calculate (Δx).

6. The angular momentum of a particle is defined as a vector $\underline{L}$, such that $\underline{L}=\underline{r}\times\underline{p}$, where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle with respect to a fixed origin O.

(a) Write down the Cartesian components $\hat{L}_x, \hat{L}_y, \hat{L}_z$ of the angular momentum operator.

(b) Show that $\left[\hat{L}_y,\hat{L}_z\right]=i\hbar\hat{L}_x$ and $\left[\hat{L}^2,\hat{L}_x\right]=0$

(c) Obtain the angular momentum operator in spherical polar coordinates (r,θ,ϕ). You may use $\underline{\hat{\theta}}=\cos\theta\cos\phi\ \underline{i}+\cos\theta\sin\phi\ \underline{j}-\sin\theta\ \underline{k}$ and $\underline{\hat{\phi}}=-\sin\phi\ \underline{i}+\cos\phi\ \underline{j}$