The Open University of Sri Lanka
B.Sc/ B.Ed Degree Programme
Final Examination - 2012/2013
Applied Mathematics – Level 05
APU3244–Graph Theory

Duration: - Three Hours

Date: - 19-12-2013. **Time:- 1.30 p.m. – 4.30 p.m.**

Answer FIVE questions only.

(01) (a) Define a regular graph. Draw a complete bipartite graph that is regular of degree 2.

(b) Determine whether each of the following graphs is regular or not:

$(\alpha)\ K_4$ $(\beta)\ K_5$ $(\gamma)\ K_{2,3}$ $(\delta)\ K_{3,3}$

Hence,

(i) state that for which values of m and n is $K_{m,n}$ regular.

(ii) deduce a general statement about the regularity of complete graphs.

Show, by giving a counter example, that the inverse of the above statement is false.

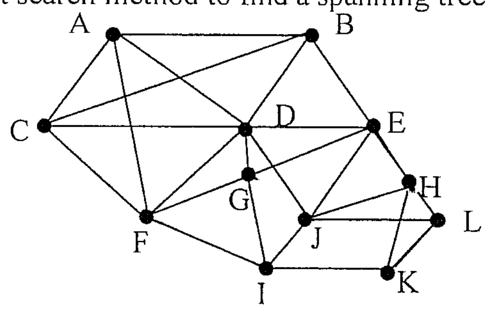
(c) Suppose that G has 7 vertices and every vertex has degree at least 3.
Is G a regular of degree 3? Justify your answer.

(02) (a) State, briefly, the breadth first search method.

(b) Find the spanning tree of the complete graph K_5 by applying the breadth first search algorithm.

Hence deduce the general type of the spanning tree of the complete graph K_n.

(c) Use the breadth-first search method to find a spanning tree by choosing 'D' as the root.



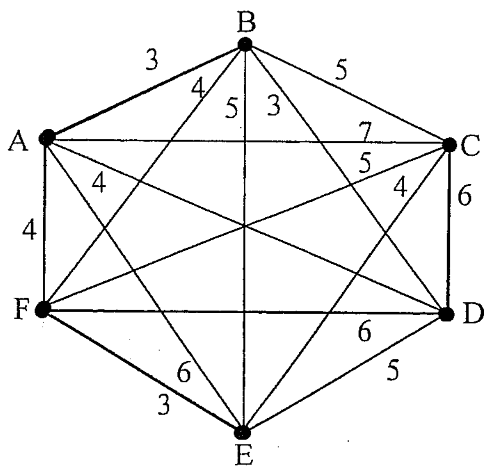
G_1

Find a spanning tree of maximum height of the above graph by using the depth-first search method.

(03) (a) Discuss, briefly, the method of solving the travelling salesman problem.

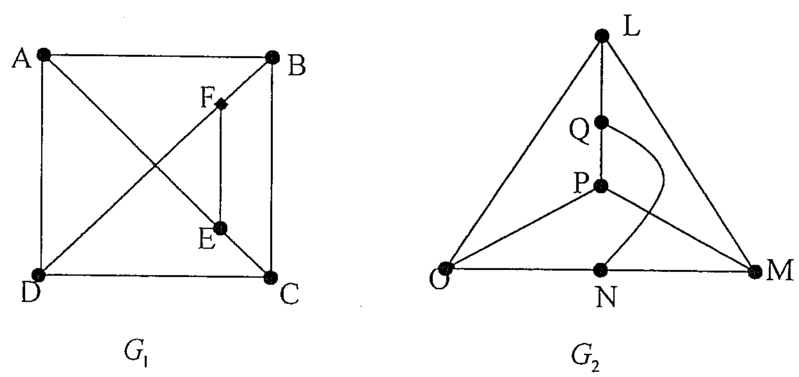
(b) Let U, V and W be three distinct vertices of a weighted graph G and let $d(U, V)$ be the distance between U and V. Show that any solution to the travelling salesman problem for G has weight at least $d(U, V)+d(V, W)+d(W, U)$.

(c) The following weighted graph shows the travelling salesman problem of six cities A, B, C, D, E and F, and the distances between the cities.

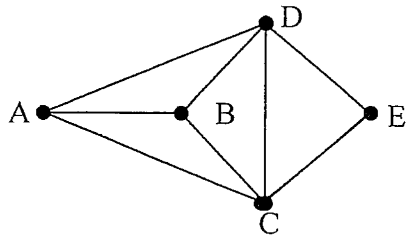


Draw the Hamiltonian cycle with least weight of the above graph, starting with city A. Hence, solve the travelling salesman problem.

(04) (a) Show that the graph G_1 is isomorphic to the graph G_2.



(b) If a graph G is $k-colorable(v)$ then its vertices can be colored with k colors so that adjacent vertices have different colors and if G is $k'-colorable(f)$ then its faces can be colored with k' colors so that no two faces with a boundary edge in common have the same color.



G

(i) Find the values of k and k' of the above graph G.

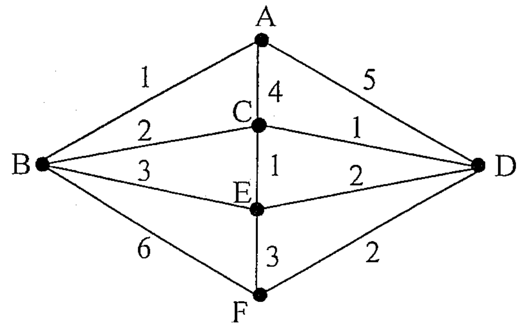
(ii) Draw the dual graph G^* of G.

(iii) Is G a self dual? Justify your answer.

(iv) Find the values of k and k' of the dual graph G^*.

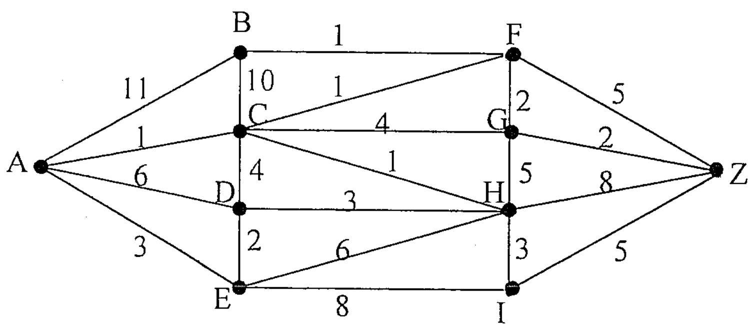
Hence, determine the relationship between the value k' of G and the value k of G^*?

(05) (a) Construct the semi- Eulerian path from A to F of the following Chinese postman problem.



Find the minimum weight from F to A and hence solve the problem.

(b) Use Dijkstra's Algorithm to find the minimal walk from A to Z of the following graph.



What is the weight $d(A,Z)$ of that minimal walk?

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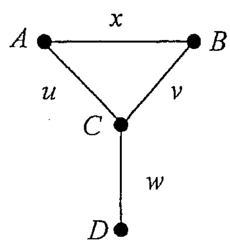
06. (a) Let A and B be any two of n vertices of a graph $G(V, E)$. Let $\deg(A_i)$ be the degree of a vertex A_i for $i = 1, 2, \ldots, n$. Define the degree of the point $x = AB$ in the line graph $L(G)$.

(b) Draw the line graph $L(K_{1,3} + x)$ of the following graph:



(i) Find the degrees of the points u, v, w, x in $L(K_{1,3} + x)$.

(ii) Draw $L^2(K_{1,3} + x)$.

(iii) Verify that the number of edges in $L(K_{1,3} + x)$ is $|E'| = \frac{1}{2}\sum_{i=1}^{n} \deg^2(A_i) - |E|$.

(iv) Show that $K_{1,3}$ is not a line graph.

(v) Is $K_4 - x$ a line graph? Justify your answer.

07. Define a digraph and a strongly connected digraph.

Let $d(x, y)$ be the minimum length among all $x - y$ walks in a digraph D. The following table gives the adjacency list of a digraph $D_1 = (D(V_1), D(A_1))$.

Vertex	Adjacent Edge
t	w
u	x , t
v	t, z
w	z, v
x	y, w
y	v, x , z
z	-

(i) Draw the digraph D_1,

(ii) Find $d(u, a)$ for all $a \in V_1$,

(iii) Find $d(a, z)$ for all $a \in V_1$,

Hence, deduce that D_1 is not strongly connected.

(iv) Let $D_2 = (D(V_1), D(A_2))$, where $A_2 = A_1 \cup \{(z, u)\}$.

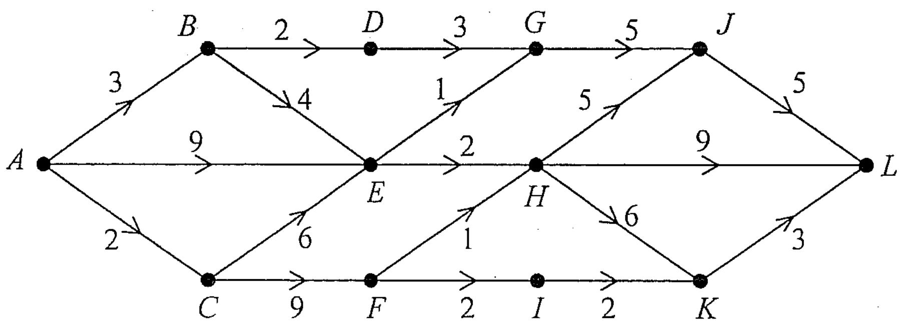
Show that D_2 is strongly connected.

(v) Is D_2 a tournament? Justify your answer.

08. (a) Let $G(V,E)$ be a directed graph of order n and size m and let $V = \{v_1, v_2, ..., v_n\}$.

State the Handshaking Dilemma.

(b) Let the following graph represents a construction of a complete house, where A and L represent the beginning and the completion of the job. Assume that the entire job cannot be completed until each path from A to L has been traversed.



(i) Obtain the critical path from A to L.

Hence find the minimum time required to finish the job E.

(ii) Verify the Handshaking Dilemma.

(iii) Is $\sum_{i=1}^{n} in\deg(v_i)^2 = \sum_{i=1}^{n} out\deg(v_i)^2$? Justify your answer.

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