The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination- 2012/2013
Applied Mathematics-Level 05
APU3150/AMU3181/AME5181 – Fluid Mechanics

Duration:-Two hours

Date:-30.11.2013 Time:-9:30a.m.-11:30a.m.

Answer **FOUR** questions only.

1. Obtain the equation of continuity for an incompressible fluid in the form $div\,\underline{q} = 0$ $\underline{q}$ being the velocity.

In terms of **cylindrical polar coordinates** (r,θ,z), the velocity potential ϕ in an irrotational motion of an incompressible fluid is given by $\phi = U\left(r+\frac{a^2}{r}\right)\cos\theta$, $r \geq a$, where U and a are positive constants. **Derive** the velocity at any point in the region of flow, in component form $\mathbf{q} = q_r\mathbf{e}_r + q_\theta\mathbf{e}_\theta$, and show that the equation of continuity is satisfied by this velocity.

Verify that the cylinder $r = a$ is a boundary of the fluid, and find the velocity there. Describe the flow at infinity, as $r \to \infty$.

Obtain the equations of the streamlines in the form $\psi(r,\theta) = b$ and $z = c$, where the function $\psi(r,\theta)$ is to be determined and b, c are arbitrary constants.

Verify further that (i) $(grad\phi)\cdot(grad\psi) = 0$ and (ii) $\nabla^2\psi = 0$.

2. A sphere, whose radius at time t is $R(t)$, with center O fixed, vibrates radially in an infinite incompressible liquid of constant density ρ occupying the region $r \geq a$. Verify that the motion of the liquid is irrotational with velocity potential $\phi(r,t) = \frac{R^2}{r}\left(\frac{dR}{dt}\right)$, where r is the distance of any point P in the liquid measured from O. The liquid is under the action of **no external body forces.** It extends to infinity, where it is at rest and the pressure there is p_∞. Assuming ***Bernoulli's equation*** show that the pressure at the surface of the sphere $(r = R)$ at time t is $p = p_\infty + \rho\left[R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2\right]$.

Given further that $R = a + b\sin(\omega t)$, where a, b, ω are constants such that $a > b > 0$, show that there will be no *cavitation* on the surface of the sphere if $p_\infty \geq (a+b)\rho\omega^2 b$.

3. A rigid sphere, of radius a is moving without rotation, so that its centre O moves in a straight line, with constant velocity V through an infinite liquid of constant density ρ, at rest at infinity. **Spherical polar coordinates** (r,θ,ω) of a **fixed point P** in the liquid are defined so that the origin coincides with O, and the axis $\theta = 0$ is along the line of motion of O.

Derive the velocity potential in the form $\phi = \dfrac{Va^3x}{2r^3}$, where $x\ (= r\cos\theta)$ is the distance of P measured along the axis $\theta = 0$, so that $\dfrac{dx}{dt} = -V$.

Deduce that the speed q is given by $q^2 = \left(\dfrac{Va^3}{2r^3}\right)^2 \left(3\cos^2\theta + 1\right)$ and that

$\dfrac{\partial\phi}{\partial t} = \dfrac{V^2a^3}{2r^3}\left(3\cos^2\theta - 1\right)$.

Given that the pressure at infinity is p_∞, show that the pressure at any point on the surface of the sphere can be expressed as $p = p_0 + C\cos^2\theta$, where the values of constants p_0 and C are to be determined.

What is the resultant thrust on the sphere? Justify your answer.

4. **Derive** the velocity potential ϕ_0 at a point P whose **spherical polar coordinates** are (r,θ,ω), due to a uniform stream $-U\underline{\mathbf{i}}$, where $\underline{\mathbf{i}}$ is a unit vector along the axis $\theta = 0$.

Show that the velocity potential ϕ_1 at the point P due to an isolated doublet of vector moment $\mu\underline{\mathbf{i}}$ placed at the origin O, is given by $\phi_1 = \mu\left(\dfrac{\cos\theta}{r^2}\right)$, fluid being at rest at infinity.

In the fluid motion represented by the velocity potential $\Phi = \phi_0 + \phi_1$, find the velocity components, and show that there is no flow of fluid across a certain spherical surface $r = a$, to be determined in terms of U and μ.

Taking this surface $r = a$ as a rigid boundary, re-write the expressions for the velocity components, involving the constants U and a (eliminating μ).

Locate the points on the sphere where the pressure takes *greatest* and *least* values, and show that these two values differ by an amount $\dfrac{9\rho U^2}{8}$.

5. Obtain the relationship between the *velocity potential* ϕ and the *stream function* ψ representing the same fluid motion, stating the conditions under which they exist.

Verify that the velocity potential $\phi = -m\log\left(\frac{r_1}{r_2}\right)$, where $r_1^2 = (x-a)^2 + y^2$ and $r_2^2 = (x+a)^2 + y^2$, m and a being constants, represents a possible motion.

Find the *components of velocity* and show that,

(i) the fluid speed, $q = \dfrac{2am}{r_1 r_2}$ and

(ii) the stream function may be expressed in the form, $\psi = -m\tan^{-1}\left(\dfrac{2ay}{x^2+y^2-a^2}\right)$.

Hence, show that the streamlines belong to a family of co-axial circles.

6. Find the complex potential w_1, at a point $P(z = x+iy)$ due to a two-dimensional uniform stream $-U\underline{\mathbf{i}}$, where $\underline{\mathbf{i}}$ is the unit vector along the positive Ox-axis.

Write down the complex potential w_2, for a two-dimensional, isolated doublet of strength μ, whose axis makes an angle α with the positive Ox-axis, placed at point $P_0(z = z_0)$, when no boundary is present.

A two-dimensional doublet of strength $\mu\underline{\mathbf{i}}$ is placed at the point $z = ia$ in a stream of velocity $-U\underline{\mathbf{i}}$ in a semi-infinite liquid of constant density occupying the region $y \geq 0$ with $y = a$ as a rigid boundary. Show that

(i) the complex potential of the motion is $w(z) = Uz + \dfrac{2\mu z}{(z^2+a^2)}$,

(ii) there are **no stagnation points** on the boundary, if $0 < \mu < 4a^2U$,

(iii) pressure on the boundary is least at the origin and greatest at the points $z = \pm a\sqrt{3}$.
