The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2012/2013

APU2144/APE4144- Applied Linear Algebra and Differential Equations

APPLIED MATHEMATICS-LEVEL 04

Duration: Two Hours.

Date: 17.12.2013	**Time: 09.30 a.m. - 11.30 a.m.**

Answer FOUR questions only.

1. (a) Define each of the following terms:
   - (i) Inverse of a matrix,
   - (ii) Rank of a matrix,
   - (iii) Equivalent matrices.

   (b) Find the inverse of the matrix A where

$$A = \begin{pmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{pmatrix}.$$

   (c) Find the rank of the matrix $\boldsymbol{B}$ where $B = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{pmatrix}.$

   (d) Find the values of the constants a and b for which the following system has

   (i) no solution,

   (ii) a unique solution,

   (iii) infinitely many solutions,

$$
\begin{aligned}
2x + 3y + 5z &= 9,\\
7x + 3y - 2z &= 8,\\
2x + 3y + az &= b.
\end{aligned}
$$

2. (a) The characteristic equation of a square matrix A of order n is $\lambda^{n+1} - \lambda = 0$. Prove that

$$\frac{2I}{2I - A} = I + \frac{1}{2^n - 1}\left(2^{n-1}A + 2^{n-2}A^2 + \ldots 2A^{n-1} + A^n\right)$$

where I is the unit matrix of order n.

(b) Transform (reduce) the following quadratic form to canonical form by an orthogonal transformation and state the corresponding modal matrix:

$$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_1x_3 - 2_1x_2.$$

3. (a) Find the general solution of each of the systems of simultaneous differential equations, given below:

(i)
$$
\begin{aligned}
\dot{x}_1 &= x_1 + x_2 - x_3\\
\dot{x}_2 &= 2x_1 + 3x_2 - 4x_3\\
\dot{x}_3 &= 4x_1 + x_2 - 4x_3,
\end{aligned}
$$

(ii)
$$
\begin{aligned}
\dot{x}_1 &= 3x_1 - x_2 + 2e^{2t}\\
\dot{x}_2 &= 4x_1 - x_2 - 2,
\end{aligned}
$$

(iii)
$$
\begin{aligned}
\ddot{x}_1 &= -x_1 - x_2\\
\ddot{x}_2 &= 4x_1 + 3x_2.
\end{aligned}
$$

4. (a) Find a sinusoidal solution of the following system of equations:

$$
\begin{aligned}
\ddot{x}_1 &= 8x_1 - 5x_2 + \sin 2t\\
\ddot{x}_2 &= 10x_1 - 7x_2 + 2\cos 2t.
\end{aligned}
$$

(b) Find the general solution of each of the differential equations given below:

(i) $3x^2 \dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 2y = 0,$

(ii) $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = \log x, \ \ x > 0.$

5. (a) A change in coordinates from (x, y) to (ζ, ϕ) is defined by $\zeta = x^2 + y$ and $\phi = x^2 - y$. Given that u is a function of the two variables x and y, find $\dfrac{\partial^2 u}{\partial x^2}$ and $\dfrac{\partial^2 u}{\partial y^2}$ in terms of partial derivatives of u with respect to ζ and ϕ.

   (b) Hence show that the second order partial differential equation $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{1}{x}\dfrac{\partial u}{\partial x} - 4x^2\dfrac{\partial^2 u}{\partial y^2} = 0$ $(x \neq 0)$ reduces to $\dfrac{\partial^2 u}{\partial \zeta \partial \phi} = 0$, when the variables are changed to ζ and ϕ.

6. By using the method of characteristics, find the general solution of each of the following differential equations:

   (a) $(x + y)\left(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right) = u,$

   (b) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2u.$