The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2012/2013
Applied Mathematics
APU 2142/ APE4142 – Newtonian Mechanics I

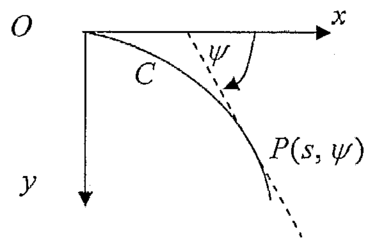
Duration :- Two Hours

Date :-05.12.2013 **Time:-01.30 p.m. 03.30 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation show that the velocity and acceleration components in intrinsic coordinates are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$ where $\underline{t}$ is the unit vector in the direction of the tangent and $\underline{n}$ is the unit vector in the direction of the inward normal.

(ii) The figure shows a curve C that forms the vertical cross-section of a smooth surface. A particle P moves in a vertical plane along the curve C, whose intrinsic equation is $s = a\tan\psi,\quad 0 < \psi < \frac{\pi}{2}$.



The coordinates (s, ψ) of P are measured relative to a fixed point O and a fixed horizontal line Ox. The particle is released from rest from the point where $\psi = \frac{\pi}{3}$ and slides down the surface along C.

(a) Show that, while the particle remains in contact with the surface, the speed v of the particle is given by $v^2 = 2ga(\sec\psi - 2)$.

(b) Show that the particle leaves the surface after it has travelled along the curve a distance $(\sqrt{15} - \sqrt{3})a$.

2. (i) With the usual notation show that the velocity and acceleration components of a particle moving in a plane in plane polar coordinates are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta .$$

(ii) The only force acting on a body, which is of mass m and is at a distance r from the centre of the Earth, is directed towards the centre of the Earth and is of magnitude $\frac{\mu m}{r^2}$, where μ is a constant.

(a) Show that the speed of a satellite of mass m moving in a circular orbit of radius a about the centre of the Earth is $\sqrt{\mu/a}$.

(b) A second satellite, of mass *3m,* is moving in the same circular orbit as the first but in the opposite direction and the two satellites collide and coalesce to form a single composite body. Show that the subsequent motion of the composite body is governed by the two equations: $r^2\dot{\theta} = \sqrt{\frac{a\mu}{4}}$, $\ddot{r} = \frac{\mu(a-4r)}{4r^3}$ where (r, θ) are the polar coordinates of the body with the centre of the Earth as pole.

(c) Find the values of r when $\dot{r}^2 = 0$.

3. (i) A particle P moves with speed V acceleration μ/r^2 directed towards a fixed point S, where $r = SP$. Prove that its orbit is an ellipse, a parabola or a hyperbola according as $V^2 \lesseqgtr 2\mu/r$, where V is the speed of the particle.

(ii) A particle P of mass m describing a circle under a force of attraction μ/r^2 directed towards the centre collides and coalesces with a particle of the same mass m which is at rest. If the composite particle moves with same law of force, show that it will describe an ellipse and show that the eccentricity is 3/4.

4. (i) Let $\underline{H}$ be the angular momentum about O of a system of particles in motion and let $\underline{M}$ be the total moment about O of the external forces acting on the system. Show that $\frac{d\underline{H}}{dt} = \underline{M}$.

(ii) A uniform rod AB of mass m and length $8a$ is free to rotate in a vertical plane about a smooth fixed horizontal axis through the point C of the rod where $AC = 2a$. The rod is initially at rest with A vertically below C but is then slightly disturbed and begins to rotate.

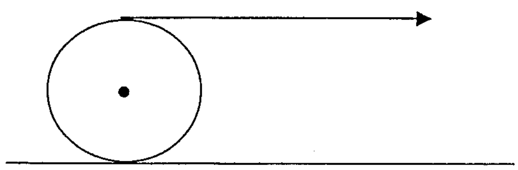
(a) Find the angular speed when the rod has turned through an angle θ.
(b) Find the magnitude of the force on the axis when the rod is vertical with A vertically above C.

5. (i) For a system of particles subjected to impulsive forces, show that the impulsive moment of the resultant force about the axis is equal to the gain of angular momentum.

(ii) A uniform rod AB of length $4a$ and mass m is free to rotate in a vertical plane about a smooth, fixed, horizontal axis through the point C of the rod where $AC = a$. The rod is released from rest with AB horizontal. When the rod first becomes vertical, end B strikes a stationary particle of mass m which adheres to the rod. Find, in terms of g and a, the angular speed of the rod after the impact and calculate the angle between the rod and the downward vertical when the rod comes to instantaneous rest.

6. (a) Show that the angular momentum of a rigid body about the axis through G perpendicular to the plane, is given by , $\underline{H}_G = I_G\,\underline{\omega}$, where $I_G = \sum_{i=1}^{n} m_i r_i'^2$ is the moment of inertia about the axis through G perpendicular to the plane.

(b) A uniform solid cylinder of mass 8kg and radius 0.25m has a string attached to a point of its surface and wound several times around the cylinder. The cylinder rests with its curved surface on a rough horizontal plane and the string is pulled in a direction parallel to the plane and perpendicular to the axis of the cylinder as shown in the diagram.



When the tension in the string is 30N the cylinder rolls along the plane. Determine the minimum value of the coefficient of friction between the cylinder and the plane.