The Open University of Sri Lanka
B.Sc/ B.Ed Degree Programme
Final Examination - 2012/2013
Applied Mathematics – Level 04
AMU2185/AME4185–Numerical Methods I

Duration: - Two Hours

Date: - 04-12-2013. **Time:- 1.30 p.m. – 3.30 p.m.**

Answer FOUR questions only.

(01) (a) Define each of the following terms :

(i) absolute error,
(ii) relative error,
(iii) percentage error.

(b) $$\int_0^{1/4} e^{x^2}\,dx = \int_0^{1/4}\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+\ldots\right)dx$$

You are given the approximation for the above value as $\overline{p}$ by ignoring the terms which are greater than x^6,

$$\int_0^{1/4} e^{x^2}\,dx \simeq \int_0^{1/4}\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}\right)dx = \overline{p}$$

Find $\overline{p}$ and compare your answer with the true value $p = 0.2553074606$ by finding absolute error and relative error.

(c) You are given that $f(x)=0$ has one root in the interval (0, 1) where,

$$f(x) = 0.1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} + \ldots$$

Find this root correct up to five decimal places using the Newton-Raphson method.

(02) (a) Consider the following two rearrangements of the equation $\ln x - 2x + 5 = 0$,

$$x = e^{2x-5} = h(x) \text{ and } x = \frac{\ln x + 5}{2} = g(x)$$

Use simple iterative method to find solutions $h(x)$ and $g(x)$.

[Hint: Use Simple iterative method and there appears to be a solution around $x = 3$.]

(b) Estimate the number of iterations that will be required to find the root of $\ln x - 2x + 5 = 0$ correct up to 6 decimal places convergence by means of

(i) Method of bisection,
(ii) Simple iterative method.

(Note: $0 \le \ln x - 2x + 5 < 4$)

(c) Comment about results of part (b)?

(03) (a) (i) Show that the equation $f(x) = e^x - 3x$ has a root in the interval [0,1].
(ii) Using the bisection method find the root of $f(x)$ defined in part (i) correct up to 3 decimal places.

(b) A root of the equation $2x - \log_{10} x = 7$ lies between 3.5 and 4.0. Using the method of false position, find this root correct up to five decimal places.

(04) (a) Derive the Newton's forward interpolation formula for a polynomial by using shift operator E.

(b) Find the polynomial $f(x)$ of degree three which takes the values of following table, using Newton's forward formula.

k	0	1	2	3
x_k	4	6	8	10
y_k	1	3	8	20

Hence determine the value of $f(2.5)$.

(05) (a) Compute the missing values of y_k and Δy_k denoted by $\boxed{?}$ in the following difference table.

y_k	?		?		?		6		?		?		?
Δy_k		?		?		5		?		?		?	
$\Delta^2 y_k$			1		4		13		18		24		

(b) Construct Newton's backward difference interpolation polynomial that goes through the following points. $f(-1)=0.1000000$, $f(-0.75)=-0.071812$, $f(-0.5)=0.024750$, $f(-0.25)=0.334938$, $f(0)=1.101000$.
Hence find an approximate value of $f(x)$ when $x=-1/3$.

(06) (a) Define $y=p(x)$, the Lagrange's polynomial of degree n through the $(n+1)$ points $(x_i, y_i); i=0,1,\ldots,n$.

(b) Find the interpolating polynomial $P(x)$ through the points $(0,3),(1,2),(2,7)$ and $(4,59)$. Hence find an approximate value when $x=3$.

(c) What is the maximum error $E(x)$ of the polynomial $P(x)$ in $(0,4)$.

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