The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 04
Final Examination – 2012/2013
Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

Duration :- Two and Half Hours

Date :- 05-12-2013 **Time :-1.30 p.m. – 3.30 p.m.**

Answer Four Questions Only.

1. A light aircraft of total mass m has an engine which can develop maximum power P. When the aircraft is moving on a level runway at speed v the total resistance to motion is mkv^2, where k is a constant. In order to take off, the pilot holds the aircraft stationary at one end of the runway and releases the brakes when the engine reaches the full power. The aircraft moves a distance x to reach a speed v and the speed required for take-off is V_0.

   (a) Show that the equation of motion of the aircraft before take-off can be written in the form $\frac{dv}{dt}=\frac{k(A^3-v^3)}{v}$, where $A^3=\frac{P}{mk}$.

   (b) Hence show that $x=\frac{1}{3k}\ln\left(\frac{A^3}{A^3-v^3}\right)$. Give a physical interpretation of A, and show that the minimum length of runway required to take-off is $\frac{1}{3k}\ln\left(\frac{A^3}{A^3-V_0^3}\right)$.

2. A particle of mass ia attached to one end of a elastic string of natural length a and modulus of elasticity $2mg$ whose other end is fixed at O. The particle is let fall from a point A, where A is vertically above O and $OA=a$. Show that its velocity will be zero at B where $OB=3a$. Calculate the time taken to travel from A to B.

3. **(a)** Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}-\underline{u}\frac{dm}{dt}$. for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being added at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(b) A spherical raindrop of radius a m falls from rest through a vertical height h, receiving throughout the motion an accumulation of condensed vapour at the rate of k $\text{kgm}^{-1}\text{s}^{-1}$ with no vertical force other than gravity acting. Show that when it reaches the ground its radius will be $k\sqrt{(2h/g)}\left\{1+\left(ga^2/2hk^2\right)^{1/2}\right\}$.

4. (a) With the usual notation show that the velocity and acceleration components of a particle moving in a plane, in plane polar coordinates are given by $\underline{v}=\dot{r}\underline{e}_r+r\dot{\theta}\underline{e}_\theta$ and $\underline{a}=(\ddot{r}-r\dot{\theta}^2)\underline{e}_r+\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$.

(b) A small bead slides with a constant speed v on a smooth wire in the shape of a cardioid $r=a(1+\cos\theta)$. Show that the angular velocity is $(v/2a)\sec\theta/2$ and that the radial acceleration is constant.

5. (a) With the usual notation show that the equation of the orbit of a particle moving in a plane under a central force F per unit mass is given by $\frac{F}{h^2u^2}=u+\frac{d^2u}{d\theta^2}$.

(b) A particle describes the curve $r=a(1+\cos\theta)$ under a central force P directed towards the pole. Find the law of the force.

6. (a) A particle P moves with speed V acceleration μ/r^2 directed towards a fixed point S, where $r=SP$. Prove that its orbit is an ellipse, a parabola or a hyperbola according as $V^2 \lessgtr 2\mu/r$, where V is the speed of the particle.

(b) A particle P of mass m describing a circle under a force of attraction μ/r^2 directed towards the centre collides and coalesces with a particle of the same mass m which is at rest. If the composite particle moves with same law of force, show that it will describe an ellipse and show that the eccentricity is 3/4.