The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2012/2013
Applied Mathematics
APU3240/APE5240 — Numerical Methods

Duration: Three Hours

Date: 31. 05. 2013 **Time: 01.30 p.m. – 04.30 p.m.**

Answer Five Questions Only.

1. (a) Derive Newton- Raphson formula for solving the equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Using the Newton- Raphson method, find the root lies [0.1, 0.2] of $x(1-\ln x)=0.5$ correct up to four decimal places.

   (d) Derive general formula to find $\sqrt{N}$ by Newton -Raphson method where N is a positive real number. Hence find $\sqrt{12}$.

2. (a) Prove that

   (i) $\Delta = E-1$, (ii) $\nabla = 1-E^{-1}$, (iii) $\delta = E^{1/2}-E^{-1/2}$,

   (iv) $\left[\left(\frac{\Delta^2}{E}\right)e^x\right]\left[\frac{Ee^x}{\Delta^2 e^x}\right]=e^x$, (v) $\Delta \ln f(x)=\ln\left[1+\frac{\Delta f(x)}{f(x)}\right]$,

   where Δ, ∇, δ and E are the forward difference , the backward difference, the central difference and the shift operators respectively.

   (b) Derive the Gregory- Newton forward interpolation formula.

   (c) Hence, interpolate $f(22)$ corresponding to the data points (20, 12), (25, 15), (30, 20), (35, 27), (40, 39) and (45, 52).

3. (a) (i) Derive the Newton's general interpolation formula with divided differences.

(ii) Hence, find the equation of degree four passing through the points

(8, 1515), (7, 778), (5, 138), (4, 43) and (2, 3).

(b) (i) Derive the Lagrange's interpolation formula.

(ii) Find the Lagrange polynomial (f) passing through the points (3, 168), (7, 120), (9, 72) (10, 63) and determine $f(6)$.

4. (a) Derive the Simpson's One –Third Rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals then show that the error in Simpson's One –Third rule is given by $|E| < \frac{(b-a)h^4}{180}M$, where M is the numerically greater value of y_0^{iv}, y_2^{iv} ...... y_{2n-2}^{iv}.

(c) Evaluate the integral $\int_0^6 \frac{1}{1+x^2}dx$, using Simpson's One third rule. Hence find an approximate value for $\tan^{-1} 6$.

5. (a) Using the Taylor series method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, with the initial condition $y(0) = 0$ at $x = 0.1$ and $x = 0.2$.

(b) Using the Taylor series method, solve $\frac{d^2y}{dx^2} + xy = 0$, with the initial condition $y(0) = 1$ and $y'(0) = 0.5$ at $x = 0.1$ and $x = 0.2$.

6. (a) (i) Derive formula for the Picard's method to solve $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = 2 - \frac{y}{x}$ with the initial condition $y(1) = 2$.

(b) (i) Derive formula for the modified Euler's method to solve $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using the modified Euler's method, solve $\frac{dy}{dx} = y - \frac{2x}{y}$ with the initial condition $y(0) = 1$ at $x = 1.2$ and $x = 1.4$ taking $h = 0.2$.

7. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using fourth order Runge-Kutta method, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 - x^2}$ with the initial condition $y(0) = 1$ at $x = 0.2$ and $x = 0.4$.

(c) Using fourth order Runge-Kutta method, solve $\frac{d^2y}{dx^2} - y^3 = 0$, with the initial condition $y(0) = 10$, $y'(0) = 50$.

8. (a) State Milne's Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = \frac{1}{2}(1+x)y^2$, $y(0) = 1$, by Taylor series method for $x = 0.2, 0.4, 0.6$ and hence find $y(0.8)$ by Milne's Predictor – Corrector Method.