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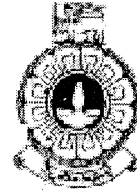
B.Sc. /B.Ed. Degree Programme

APPLIED MATHEMATICS-LEVEL 05

APU3141/APE5141- Linear Programming

Final Examination 2012/2013

**Duration: Two Hours.**



**Date: 12.06.2013**

**Time: 9.30 a.m- 11.30 a.m**

**Answer four questions only.**

- (1) a) The evening shift resident doctors in a government hospital work for five consecutive days and have two consecutive days off. Their five days of work can start on any day of the week and the schedule keeps rotating. The hospital requires the following minimum number of doctors for each day.

| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|-----|-----|-----|-----|-----|-----|-----|
| 35  | 55  | 60  | 50  | 60  | 50  | 45  |

No more than 40 doctors can start their five working days on the same day.

Formulate the problem as a linear programming problem to minimize the number of doctors employed by the hospital.

- b) The final simplex tableau for a profit maximization linear programming problem is given below. Here,  $X_1$ ,  $X_2$  are products,  $s_1$ ,  $s_2$  are the slack in labor hours and raw material and  $Z$  is the total profit:

| Basis | $X_1$ | $X_2$ | $s_1$ | $s_2$ | Solution |
|-------|-------|-------|-------|-------|----------|
| $X_1$ | 1     | 0     | 0.75  | -0.01 | 7.5      |
| $X_2$ | 0     | 1     | -0.50 | 0.01  | 5        |
| $Z$   | 0     | 0     | 5     | 0.10  | 550      |

- (i) Is the solution feasible? Justify.
- (ii) Is the solution optimal? Justify.
- (iii) How many of  $X_1$  and  $X_2$  products are needed to produce according to this solution? What is the total profit?

(2) Use Big-M method to solve the following linear programming problem:

$$\text{Maximize } z = -x_1 + 3x_2,$$

$$\text{Subject to } x_1 + 2x_2 \geq 2,$$

$$3x_1 + x_2 \leq 3,$$

$$x_1 \leq 4,$$

$$x_1 \geq 0, x_2 \geq 0.$$

(3) A taxi hire company has one taxi at each of five depots a, b, c, d and e. A customer requires a taxi in each town, namely A, B, C, D and E. Distance (in kms) between depots (origins) and towns (destinations) are given in the following distance matrix:

| Origin \ Destination | a   | b   | c   | d   | e   |
|----------------------|-----|-----|-----|-----|-----|
| A                    | 140 | 110 | 155 | 170 | 180 |
| B                    | 115 | 100 | 110 | 140 | 155 |
| C                    | 120 | 90  | 135 | 150 | 165 |
| D                    | 30  | 30  | 60  | 60  | 90  |
| E                    | 35  | 15  | 50  | 60  | 85  |

How should taxis be assigned to customers so as to minimize the distance travelled?

(4) An organization has four destinations at  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  and three sources at  $S_1$ ,  $S_2$  and  $S_3$  for supply of a good. The transportation cost per unit is given below.

|                      | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Availability (Supply) |
|----------------------|-------|-------|-------|-------|-----------------------|
| $S_1$                | 13    | 16    | 19    | 17    | 250                   |
| $S_2$                | 17    | 19    | 16    | 15    | 200                   |
| $S_3$                | 15    | 17    | 17    | 16    | 250                   |
| Requirement (Demand) | 100   | 150   | 250   | 100   |                       |

- a) Formulate a linear programming model for the given transportation problem.
- b) Find an initial basic feasible solution to this problem by Northwest corner rule.
- c) Find the optimal solution.

(5) a) What is meant by dual problem of a linear programming model?

b) Consider the following linear programming problem:

$$\text{Minimize } z = y_1 + 2y_2,$$

$$\text{Subject to } y_1 + 2y_2 \leq -3,$$

$$y_1 + 3y_2 \leq -1,$$

$$y_1, y_2 \geq 0.$$

- (i) Give the dual linear programme for the above problem.
- (ii) Solve the dual linear programme given in (i) by using the dual simplex method. Hence, write the solution of the primal problem.

(6) Solve the following linear programming problem using two phase simplex method:

$$\text{Minimize } z = 4x_1 + 6x_2 + 5x_3,$$

$$\text{Subject to } 2x_1 + 4x_2 + 3x_3 \geq 32,$$

$$x_1 + 2x_2 + 4x_3 \geq 28,$$

$$x_1, x_2, x_3 \geq 0.$$

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