The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2012/2013
AMU3187/ AME 5187- Mathematical Methods II
Applied Mathematics -Level 05

**Duration: Two Hours.**

| **Date: 26.06.2013** | **Time: 1.00 p.m. - 3.00 p.m.** |
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**Answer FOUR questions only.**

1. (i) Sketch the graphs of each of the functions defined by

   (a) $f(x) = \sin x, \quad 0 \le x < \pi, \quad f(x+\pi) = f(x),$

   (b) $f(x) = \dfrac{1}{p}x, \quad 0 \le x < p, \quad f(x+p) = f(x).$

   (ii) Find the half range expansions of the function

$$f(x) = \begin{cases} 2x & 0 < x < 1/2 \\ 2(1-x) & 1/2 \le x \le 1. \end{cases}$$

2. (i) Define the orthogonality of two real valued functions $f_m(x)$ and $f_n(x)$ which are defined on an interval $a \le x \le b$.

   (ii) In each of the following cases, show that the given set is orthogonal on the given interval and determine the corresponding orthonormal set:

   (a) $1, \cos x, \cos 2x, \cos 3x, \ldots; \qquad 0 \le x \le 2\pi$

   (b) $\sin x, \sin 2x, \sin 3x, \ldots; \qquad 0 \le x \le \pi$

(iii) Determine the constants $\alpha_0$, $\beta_0$, $\beta_1$, $\gamma_0$, $\gamma_1$ and $\gamma_2$ so that the functions

$g_0 = \alpha_0$, $g_1 = \beta_0 + \beta_1 x$ and $g_2 = \gamma_0 + \gamma_1 x + \gamma_2 x^2$

form an orthonormal set on the interval $-1 \le x \le 1$.

3. Given the boundary value problem

$$\frac{d^2 y}{dx^2} + \mu y = 0$$
$$y(-1) = y(1), \quad y'(-1) = y'(1).$$

(i) Show that it is a Sturm-Liouville problem.

(ii) Find the eigenvalues and eigenfunctions.

(iii) Verify that the eigenfunctions are mutually orthogonal in the interval $-1 \le x \le 1$.

4. The Legendre polynomial of degree $n$, $P_n(x)$, is given by the expansion

$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n$, for sufficiently small $|t|$.

Using this expansion prove that

(i) $nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$,

(ii) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$,

(iii) $(n+1)P_{n+1}(x) + nP_{n-1}(x) = (2n+1)xP_n(x)$,

(iv) $\|P_n(x)\| = \sqrt{\dfrac{2}{2n+1}}$.

5. Let $J_n(x)$ be the Bessel function of order $n$ given by the expansion

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n .$$

Verify each of the following identities when $n$ is a positive integer.

(i) $J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$

(ii) $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$

(iii) $J'_n(x) + \frac{n}{x} J_n(x) = J_{n-1}(x)$

(iv) $\frac{d}{dx}[J_n^2(x)] = \frac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)]$

6. Solve the Laplace's equation,

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial r^2} = 0$$

in the disc $0 < r < c$ if the boundary condition is

$u(c, \theta) = |\theta|$ ; $\quad -\pi < \theta < \pi$ .