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The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination-2012/2013
Applied Mathematics – Level 05
AMU3182 /AME5182 – Mathematical Methods-I

Duration: - Two Hours

Date: 12th June 2013 **Time: 1.00 p.m. - 3.00 p.m.**

Answer Four Questions Only.

01 (a) Find the general solution of the simultaneous differential equations:

$$\dot{x}_1 = x_1$$
$$\dot{x}_2 = x_2 - x_3$$
$$\dot{x}_3 = x_2 + x_3.$$

where the dot (.) denotes the derivative with respect to a variable t .

(b) Using (a) or otherwise, write down the general solution of the following system:

$$\ddot{x}_1 = x_1$$
$$\ddot{x}_2 = x_2 - x_3$$
$$\ddot{x}_3 = x_2 + x_3.$$

(c) Consider the first order system:

$$\dot{x} = -2tx + 3y^2$$
$$\dot{y} = -3x^2(1 - y)$$

with the initial conditions $x(0) = -1$ and $y(0) = 2$.

Use Euler method with step size $h = 0.1$ to compute approximations for $x(t)$ and $y(t)$ at time $t = 0.1$ and $t = 0.2$.

02 (a) Solve each of the following problems:

(i) $u'(x) + u(x) = 0; \quad u(0) = 1,$

(ii) $x^2 u''(x) + xu'(x) + u(x) = 0; \quad (x > o),\ u(1) = 0,\ u(e) = \sin(1),$

(iii) $(1 - x^2)u''(x) - xu'(x) + 4u(x) = 0 \quad (0 < x < 1) \quad u(0) = 0, u'(0) = 1.$

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(b) Suppose $u(x,y) = f(y^2 - x) + g(y^2 + x)$ is the general solution of a partial differential equation in the region $y > 0$. Find the particular solution which satisfies the additional conditions $u(x,y) = 1 + x^2, \dfrac{\partial u}{\partial y}(x,1) = 2x$.

03 (a) Find the general solution $w = w(x,y)$ of the partial differential equation:

$$\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial w}{\partial x} = 0$$

(b) The function $u(x,y)$ satisfies the partial differential equation:

$$3x^2 \frac{\partial^2 u}{\partial y^2} - \frac{1}{12}\frac{\partial^2 u}{\partial x^2} - 6x^2 \frac{\partial u}{\partial y} + \left(x + \frac{1}{12x} \right) \frac{\partial u}{\partial x} = 0 \ (x \neq 0).$$

(i) Classify the equation as hyperbolic, parabolic or elliptic.

(ii) Show that characteristic coordinates may be chosen to be $\varsigma = y - 3x^2$ and $\phi = y + 3x^2$.

(iii) Use the characteristic coordinates and the chain rule to transform the above partial differential equation to its standard form.

(iv) Hence, using the result of part (a), find the general solution $u = u(x,y)$ of the equation.

04 (a) Show that Eigen value problem

$X''(x) + \lambda X(x) = 0 \ (0 < x < \pi)$ with

$X'(0) = X(\pi) = 0$

has Eigen values $\lambda_n = \dfrac{(2n-1)^2}{4}$ and corresponding eigen function

$X_n(x) = \cos\left(\dfrac{2n-1}{2}\right)x \quad (n = 1,2,3,...)$.

(b) Consider the following differential equation

$(4x+1)^2 u''(x) + 2(4x+1)u'(x) - u(x) + \lambda u(x) = 0$ with the boundary conditions $u(0) = 0$ and $u(1) = 0$.

By using the change of variable $x = \dfrac{1}{4}(t-1)$, find the general solution of the differential equation for the case $\lambda > 5$.

05 (a) Find the general solution of the following partial differential equations by using the integrating factor method.

(i) $\dfrac{\partial u(x,y)}{\partial x}+\dfrac{1}{x^2(1+y)}u=2(1-y)e^{\frac{1}{x(1+y)}}$;$(x\neq 0, y\neq -1)$,

(ii) $y\dfrac{\partial^2 u(x,y)}{\partial y\partial x}-\dfrac{\partial u(x,y)}{\partial x}=xy^2\cos(xy)$.

(b) Find the general solution of the following pair of partial differential equations:

$$\frac{\partial u}{\partial y}=5y^4x-3\cos 3x$$

$$\frac{\partial u}{\partial x}=y^5+xe^x$$

(c) Find the set of characteristic curves for each of the following differential equations,

(i) $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}-u=0;\ (x\neq 0)$,

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2}-y^2\dfrac{\partial^2 u}{\partial y^2}-u=0;\ (x\neq 0, y\neq 0)$

where u is a function of x and y only.

06 (a) Find the sinusoidal particular solution of the system of equations:

$$4\ddot{x}_1+\ddot{x}_2+3\dot{x}_1+6x_1+4x_2=2\sin 2t-\cos 2t$$
$$\ddot{x}_1+4\ddot{x}_2+3\dot{x}_2+4x_1+3x_2=\cos 2t.$$

(b) Solve the following inhomogeneous system:

$$\dot{x}_1=x_1+2x_2+6e^{3t}$$
$$\dot{x}_2=2x_1+x_2+2e^{3t}.$$