The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme- Level 04
Final Examination- 2012/2013
AMU2181/AME4181-Mathematical Modelling I
Applied Mathematics

Duration: Two Hours.

Date: 12. 06. 2013 **Time: 09.30a.m.-11.30a.m.**

Answer Four Questions Only.

1. Consider two different types of foodstuffs F_1 and F_2. Assume these foodstuffs contain vitamin V_1 and V_2. Minimum daily requirement of V_1 is 40 units and V_2 is 50 units. Food F_1 contains 2 units of vitamin V_1 and 3 units of vitamin V_2. Further, F_2 contains 4 units of V_1 and 2 units of V_2. The cost of 1 unit of F_1 is Rs. 3 and of F_2 is Rs. 2.50. Develop a linear programming model and solve it graphically to determine the minimum cost of diet that will supply the minimum daily requirement of each vitamin.

2. (a) Briefly describe the following terms of a linear programming problem:

   (i) Basic solution,
   (ii) Basic feasible solution,
   (iii) Non-basic variable.

   (b) Consider the following linear programing problem:

   Maximize $z = 4x_1 + 10x_2$

   subject to

   $$2x_1 + x_2 \leq 10$$
   $$2x_1 + 5x_2 \leq 20$$
   $$2x_1 + 3x_2 \leq 18$$
   $$x_1, x_2 \geq 0$$

   (i) Write all basic feasible solutions of the above problem distinguishing basic and non-basic variables.
   (ii) Find the optimal solution by explicitly enumerating the solutions obtained in (i).

3. Solve the following linear programming problem using simplex method.

$$\begin{aligned} \text{Maximize } & Z = x_1 + 2x_2 + x_3, \\ \text{subject to } & 2x_1 + x_2 - x_3 \le 2, \\ & -2x_1 + x_2 - 5x_3 \ge -6 \\ & 4x_1 + x_2 + x_3 \le 6, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

4. Consider the following linear programming problem:

$$\begin{aligned} \text{Maximize } & Z = 3x_1 - x_2 \\ \text{subject to} & \\ & 2x_1 + x_2 \ge 2 \\ & x_1 + 3x_2 \le 3 \\ & x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(a) Solve this problem using the big M method.
(b) What is the upper limit of the objective function coefficient of the variable x_2 for the current solution to remain optimal?

5. Consider the following linear programming problem:

$$\begin{aligned} \text{Minimize } & Z = 24x_1 + 30x_2 \\ \text{subject to} & \end{aligned}$$

$$\begin{aligned} 2x_1 + 3x_2 &\ge 10 \\ 4x_1 + 9x_2 &\ge 15 \\ 6x_1 + 6x_2 &\ge 20 \\ x_1, x_2 &\ge 0 \end{aligned}$$

What is the dual of the above problem? Solve the dual problem and hence, write the solution to the primal problem.

6. Consider the following linear programming problem:

Maximize $Z = x_1 + 4x_2$

subject to

$$\begin{aligned} x_1 + 6x_2 &\leq 24 \\ x_1 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

where x_1 and x_2 represent number of units of two products produced by a company and the constraints are corresponding to raw material and machine time.

(a) Find the optimum solution for this linear programming problem using simplex method.
(b) Find the range over which the objective function can vary for x_1 and the current solution still remains optimal.
(c) Find the interval for the raw material over which the current solution remains optimal.