00106

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX 6241 – Field Theory
Final Examination – 2009/2010

Date: 2010-03-10 | Time: 0930 – 1230 hrs.

Answer any FIVE questions.

1.

(a) A vector $\underline{A}$ is given by $\underline{A} = \underline{a}_x x + \underline{a}_y y + \underline{a}_z z$, where $\underline{a}_x$, $\underline{a}_y$ and $\underline{a}_z$ are unit vectors in x, y and z directions. $V(x,y,z)$ is a scalar function. Show that

(i) $\nabla \times (\nabla V) = 0$

(ii) $\nabla . (\nabla \times \underline{A}) = 0$

(b) The curved surface of a cylinder is given by $x^2 + y^2 = 4,\ 0 \le z \le 5$.
The surface F is defined as the portion of the above surface which lies in the first quadrant. (see Fig.1)

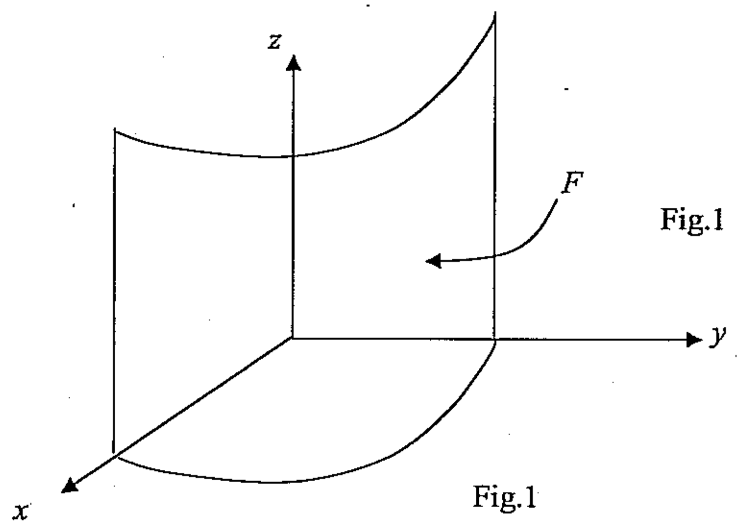

Fig.1

The surface F intercepts a magnetic field whose intensity $\underline{B}$ is given by

$$\underline{B} = \underline{a}_x z + \underline{a}_y kx - \underline{a}_z 2y^2 z,$$ where k is a constant.

(i) Calculate the total magnetic flux through the surface.

(ii) What is the answer if the field component in the y-direction is increased by a factor 2?

2.

(a) An electric field is given by $\underline{E} = \underline{a}_x 2xyz + \underline{a}_y x^2 z + \underline{a}_z x^2 y$.

(i) Show that $\underline{E}$ is a conservative field.

(ii) Can $\underline{E}$ be represented by the equation $\underline{E} = -\nabla V$? Justify your answer. ($V = V(x, y, z)$ represents a scalar function.)

(iii) Determine the work done by moving a unit charge from (1, 1, 1) to (2, 3, 4).

(b) (i) State Divergence theorem.

(ii) A vector field is given by $\underline{A} = \underline{a}_r r^2 + \underline{a}_z 2z$. Verify the Divergence theorem for the circular cylindrical region enclosed by $r = R, z = 0$ and $z = h$.

$$\left(\nabla.\underline{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z} \right)$$

3.

(a) (i) State Stokes' theorem.

(ii) A vector field is given by $\underline{F} = \underline{a}_x y^3$. S^1 is the surface of the hemisphere given by $x^2 + y^2 + z^2 = 1,\ z \ge 0$.
Using Stokes' theorem evaluate the integral $\iint_{S^1} \nabla \times \underline{F}.d\underline{S}$.

$$\left(\int_0^{2\pi} \sin^4\theta \, d\theta = \frac{3\pi}{4} \right)$$

(b) An electromagnetic wave propagates in a perfect dielectric medium. Starting from Maxwell's Curl equation for $\underline{H}$, show that the charge density ρ of the medium is unaffected by the electromagnetic field.

(c) What can you say about ρ if the medium is not a perfect dielectric?

4.

(a) Define *Poyinting* vector. What physical quantity does it represent? What is the direction of the *Poyinting* vector?

(b) In a lossless medium, the instantaneous power leaving a closed surface can be found by calculating the rate of change of stored energy.

(i) Write an expression for the stored energy density assuming that the dielectric medium to be lossless.

(ii) Write an expression for the total instantaneous power.

(iii) An antenna is located at the origin of a spherical coordinate system. The $\underline{E}$ field in free space at the coordinate point $P(r, \theta, \varphi)$ is given by

$$\underline{E} = \frac{E_0}{r} \sin\theta \sin(\omega(t - kr))\ \underline{u}_\theta$$

(α) Write an expression for the impedance of the medium assuming that the permittivity and the permeability of free space are ε_0 and μ_0 respectively.

(β) Write an expression for the $\underline{H}$ field at P.

(γ) Write expressions for electric energy density (w_e) and magnetic energy density (w_m) respectively at $r = R$.

(η) Derive an expression for the total instantaneous power radiated by the antenna.

(c) Briefly explain how you would calculate total instantaneous power of the antenna if the medium were lossy.

5.

(a) Write Maxwell's equations.

(b) Simplify the above equations if field components vary sinusoidally with angular frequency ω.

(c) $\underline{E}$ - and $\underline{H}$ field components ($\underline{E}$ and $\underline{H}$) in a simple, non-conducting, source free medium vary sinusoidally.

(i) Write expressions for $\nabla.\underline{B}, \nabla.\underline{H}, \nabla\times\underline{B}$ and $\nabla\times\underline{H}$.

(ii) Show that new field components defined by

$$\underline{E}' = \underline{E}\cos\alpha + \eta\underline{H}\sin\alpha$$

$$\underline{H}' = -\frac{\underline{E}}{\eta}\sin\alpha + \underline{H}\cos\alpha\text{, where } \eta = \sqrt{\frac{\mu}{\varepsilon}}.$$

also satisfy Maxwell's equations.

(d) (i) Modify the expression for $\nabla\times\underline{H}$ in (c) (i) if the medium has a non-zero conductivity.

(ii) Derive an expression for complex permittivity e^* and write an expression for loss tangent for (d)(i).

6.

(a) A wave is propagating in a non-conducting source free medium.

(i) Using Maxwell's equations show that

$$\nabla^2\underline{E} - \frac{1}{u^2}\frac{\partial^2\underline{E}}{\partial t^2} = 0\text{, where } u \text{ is a constant.}$$

[For any given vector $\underline{A}$, $\nabla\times\nabla\times\underline{A} = \nabla(\nabla.\underline{A}) - \nabla^2\underline{A}$]

(ii) (α) What is a plane wave?

(β) Simplify the equation in (a) (i) for a uniform plane wave whose field components vary sinusoidally.

(iii) Solve the equations you derived in (β) and show that the total E field consists of two waves propagating in the positive- and negative z-directions.

(iv) Find the value of the propagation constant if the wave is propagating in free space. Assume that the frequency of the wave is 220 *MHz*.

(b) (i) Modify equations in (a) (i) if the medium is conducting.

(ii) Find the propagation constant of a sinusoidal electromagnetic wave propagating in a *lossy* dielectric.

7.

(a) Write *Laplace* equation in Cartesian coordinates for a scalar potential $V(x,y,z)$.

(b) If the scalar potential function $V(x,y,z) = X(x).Y(y).Z(z)$ show that $X(x)$, $Y(y)$ and $Z(z)$ are the solutions of following 3 equations:

$$\frac{d^2X(x)}{dx^2} + k_x^2 X(x) = 0 \quad , \frac{d^2Y(y)}{dy^2} + k_y^2 Y(y) = 0 , \frac{d^2Z(z)}{dz^2} + k_z^2 Z(z) = 0$$

k_x, k_y and k_z are constants related by the equation $k_x^2 + k_y^2 + k_z^2 = 0$

(c) Solution to the differential equation $\frac{d^2X(x)}{dx^2} + k_x^2 X(x) = 0$ is given by

$$\begin{aligned} X(x) &= A_0 x + B && \text{if } k_x = 0 \\ &= A_1 \sin k_x x + B_1 \cos k_x x && \text{if } k_x^2 > 0 \\ &= Ce^{kx} + De^{-kx} && \text{if } k_x^2 < 0 \end{aligned}$$

You may use this result to solve the following problem.

Two parallel plane electrodes (electrodes 1 and 2) separated by a distance b are maintained at a zero potential as shown in Fig.7. The electrodes extend infinitely in the positive x- and z directions. A third electrode (electrode 3) which extends infinitely in the z-direction, kept perpendicular to the two parallel electrodes is maintained at a potential V_0.

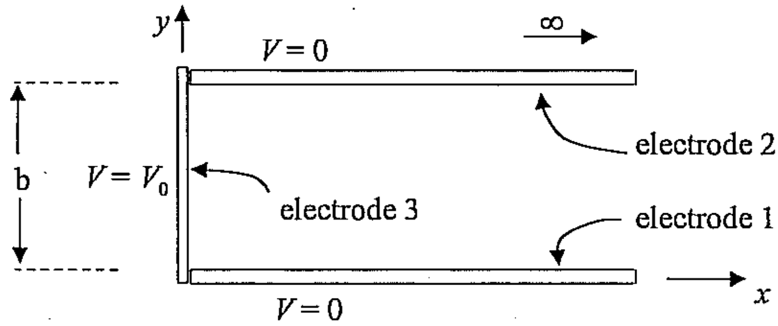

Fig.7

(i) Write boundary conditions for the potential function.

(ii) Show that

(α) $Z(z) = A$

(β) $X(x) = Be^{-kx}$

where A, B and k are constants.

8.

(a) (i) Write Maxwell's Curl equations for a source free medium.

Assuming that the field components vary sinusoidally, derive equations for E_x^0, E_y^0, H_x^0 and H_y^0 in terms of X and Y derivatives of E_z^0 and H_z^0.(Use $\gamma = \frac{\partial}{\partial z}$)

(ii) If the wave propagation is transverse magnetic, simplify the equations given in (i).

(b) To find E_x^0, E_y^0, H_x^0 and H_y^0 for *transverse magnetic* mode in a *waveguide* (Fig.8), let us try to evaluate E_z^0.

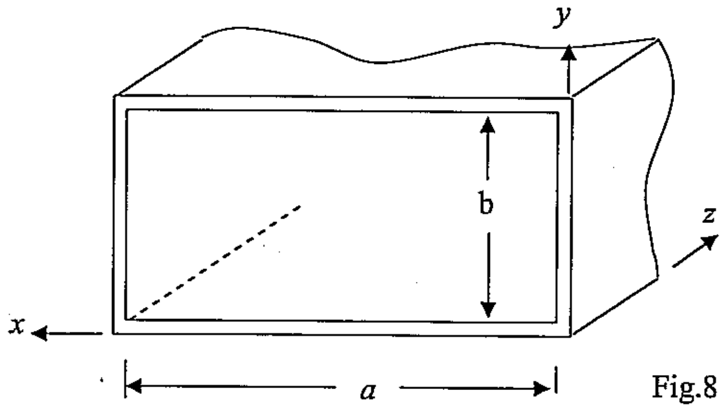

Fig.8

If we write $E_z^0 = X(x).Y(y)$, we can solve wave equation
$\nabla^2 \underline{E} = \gamma^2 \underline{E}$ for E_z^0 by substituting the expressions for E_x^0, E_y^0, H_x^0, H_y^0 and E_z^0 in the equation.

(i) If $X(x) = A_1 \cos k_x x + B_1 \sin k_x x$ and $Y(y) = C_1 \cos k_x y + D_1 \sin k_x y$

(α) write boundary conditions for E_y^0 and E_z^0 inside the waveguide.

(β) show that $E_z^0 = A \sin(k_x x).\sin(k_y y)$, where A is a constant.

(ii) Find the values of E_x^0, E_y^0, H_x^0 and H_y^0 for *transverse magnetic* mode.

(c) Why cannot *TEM* waves propagate in a waveguide?