

THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme, Continuing Education Programme

APPLIED MATHEMATICS-LEVEL 04

PCU2142/PSU2182/ PCE4142/PSE4182- DESIGN AND ANALYSIS OF EXPERIMENTS

FINAL EXAMINATION 2013/2014



**Duration: Two Hours.**

**Date: 19.11.2014**

**Time: 1.30p.m- 3.30p.m**

**Answer FOUR questions only.**

**Instructions:**

- This question paper consists of 06 questions and 05 pages.
- Statistical Tables are provided.
- Non-programmable calculators are permitted.
- Consider the level of significance as 0.05 for all the tests.

1. An experiment was run to determine whether temperature affects the density of a certain type of brick. Four temperature levels, say  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ , were tested in the experiment. The experiment led to the following data:

Temperature Level	Density				
$T_1$	21.8	21.9	21.7	21.6	21.7
$T_2$	21.7	21.4	21.5	21.4	21.6
$T_3$	21.9	21.8	21.8	21.6	21.5
$T_4$	21.9	21.7	21.8	21.4	21.5

- (a) Write a suitable statistical model for the density of the brick in terms of the effects of the temperature.
- (b) Specify the hypotheses that are to be tested.

- (c) Construct the complete analysis of variance (ANOVA) table and test your hypotheses in part (b).
- (d) Based on the decision you have taken in the above, what can you conclude about the effect of temperature on the density of the trick?
2. Road development authority studied the two characteristics, durability and visibility of five different paints, say *A*, *B*, *C*, *D*, and *E* at four locations, say 1,2,3,4 in the city. The four locations were selected, thus reflecting variations in traffic densities throughout the city. At each location, a random ordering of the five paints was employed to the chosen road surface. After a suitable period of exposure to weather and traffic, a combined measure of the both durability and visibility was obtained. The data obtained are as follows (the higher the score, the better the level of characteristics).

Location	Paint				
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
1	20	15	28	18	30
2	15	18	10	13	11
3	16	8	10	12	8
4	27	35	41	30	28

These data were given to two students, say *P* and *Q*, separately and asked to construct the ANOVA to analyze the data. They have given following results. Some values were missing in each ANOVA table and marked with “\*”.

Source	DF	SS	MS	F	P
Paint	*	*	*	*	0.116
Error	*	407.3	*		
Total	*	648.6			

Source	DF	SS	MS	F	P
Paint	*	*	*	*	0.011
Location	*	*	*	*	0.004
Error	*	136.70	*		
Total	*	648.55			

- (a) What types of experimental designs have been assumed by students  $P$  and  $Q$  in analyzing the data?
- (b) Complete each ANOVA table by computing missing values.
- (c) Among the two analyses in the above, which is the most suitable analysis for these data? Justify your answer.
- (d) Write down the model equation of the selected model. Define all the terms of the model.
3. An experiment was carried out to investigate the effect of fertilizer on the yield of a certain variety of paddy. Three amounts of fertilizer, say 10lbs, 15 lbs and 25lbs were tested along with the control (i.e. no fertilizer) using a Complete Randomized Design with 4 replicates per each treatment. The yields obtained are as follows.

Treatment	Yield (bushels/plot)			
Control	5.5	4.9	4.2	5.2
5lbs	6.1	11.2	9.0	6.0
15lbs	4.2	9.7	8.1	9.5
25lbs	16.9	13.7	16.9	15.4

A part of the ANOVA table related to this study is given below.

Sources of Variation	Sum of Squares
Treatment S.S	254.6
Error S.S	46.3
Total S.S	300.9

- (a) Identify the treatment structure of the experiment. Justify your answer.
- (b) Test whether following meaningful comparisons are orthogonal,
- Fertilizer and Control
  - Two amounts of the fertilizer
- (c) Test whether one of the comparisons in part (b) is significant. Interpret the result.

4. Suppose that you have been asked to study the effect of four assembly methods, say  $A$ ,  $B$ ,  $C$ , and  $D$  on assembly time for a television component. You are given the following information.
- There are considerable numbers of operators; who are familiar with each of 4 assembly methods.
  - The order of assembly may affect the assembly time irrespective of the assembly method.
- (a) Propose a suitable design structure for this experiment. Justify your answer.
- (b) An industrial manager conducts this experiment and the layout of the results of experiment is given below.

Order of assembly	Operator			
	1	2	3	4
1	C 10	D 14	A 7	B 8
2	B 7	C 18	D 11	A 8
3	A 5	B 10	C 11	D 9
4	D 10	A 10	B 12	C 14

Output of the above data is given below.

Source of Variation	Degrees of freedom	Sum of Squares	Mean Squares	F value
Method	3	72.5	24.2	14.2
Operator	3	51.5	17.2	10.1
Order	3	18.5	6.2	3.6
Error	6	10.5	1.7	
Total	15	153		

- i) State the hypotheses that can be tested from this analysis, and test them.
- ii) Write down your conclusions based on the results obtained.
- iii) Find the Least Significant Difference (LSD) for comparing any two treatment means.
- iv) Hence or otherwise test whether assembly methods  $A$  and  $B$  are equally effective. Clearly explain your answer.

5. Three brands of batteries are under study. It is suspected that the lives in weeks of the three brands are different. Five batteries of each brand are tested and the results are shown in the table below.

Types of batteries	Weeks of Life				
	Brand A	100	96	92	96
Brand B	76	80	75	84	82
Brand C	108	100	96	98	100

- (a) Estimate the mean life time for brand A along with its standard error.
- (b) Construct a 95% confidence interval for the difference between the mean life time of brand A and B.
- (c) Using the result of part (b) test whether there is a difference between the mean life time of brand A and B.
6. The management of a hotel is interested in studying the effects of washing machines and detergents on whiteness of bed sheets. The hotel has 2 washing machines and 2 brands of detergent. They randomly assign 3 bed sheets for each combination of machine and detergent (each sheet is only observed for one combination of machine and detergent). After washing, the sheets are measured for whiteness (high scores are better).

Washing Machine	Brands of Detergent					
	$D_1$			$D_2$		
$M_1$	24	26	25	32	34	35
$M_2$	37	40	42	47	50	55

- (a) How many treatment combinations can be tested in this experiment? What are they?
- (b) Identify the treatment structure of the experiment. Justify your answer.
- (c) Construct the complete Analysis of Variance table.
- (d) Test whether the effects of washing machine and brands of detergent on the whiteness of bed sheets are significant or not. Clearly explain your answer.

\*\*\*\*\*