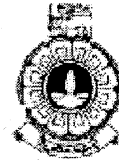


The Open University of Sri Lanka  
 B.Sc. /B.Ed. Degree Programme  
 Final Examination - 2013/2014  
 Pure Mathematics - Level 05  
 PUU3143/PUE5143 – Riemann Integration



**Duration: Two Hours**

**Date: 25.11.2014**

**Time: 1.30 p.m. – 3.30 p.m.**

**Answer Four Questions Only.**

1. (a) Let  $f$  be a bounded function on  $[a, b]$  and let  $P$  be a partition of  $[a, b]$ .  
 Prove that  $L(P, f) \leq U(P, f)$ .
  - (b) Let  $f$  be a function bounded on  $[a, b]$ . Suppose that  $P_1$  and  $P_2$  are partition of  $[a, b]$  such that  $P_2$  is a refinement of  $P_1$  and  $P_2$  has only one extra point than  $P_1$ .  
 Prove that  $L(P_1, f) \leq L(P_2, f)$ .
  - (c) Let two bounded function  $f$  and  $g$  defined on  $[0, 2]$  and  $P \in \mathcal{P}[0, 2]$ . Determine whether  $U(P, fg) = U(P, f)U(P, g)$ .
2. (a) Let  $f(x) = k$  for  $x \in [a, b]$ , where  $k$  is a constant. Prove that  $\int_a^b f(x) dx = k(b - a)$ .
  - (b) Let  $f(x) = \begin{cases} 2, & x \in [2, 3] \\ 3, & x \in (2, 4) \end{cases}$ . Show that  $\int_2^4 f(x) dx = 5$ .
  - (c) Let two function  $f$  and  $g$  defined on  $[0, 1]$ . Determine whether  $\int_0^1 (f + g)(x) dx = \int_0^1 f(x) dx + \int_0^1 g(x) dx$ .
3. (a) State Riemann's criterion.
  - (b) Let  $f(x) = x^2$ , for  $x \in [a, b]$  where  $a \geq 0$ . Use Riemann's criterion to show that  $f$  is Riemann integrable on  $[a, b]$ .
  - (c) Let  $f(x) = \begin{cases} 1, & x \in [0, 1] \cap \mathbb{Q} \\ 0, & x \in [0, 1] \cap \mathbb{Q}^c \end{cases}$ . Use Riemann's criterion to show that  $f$  is not Riemann integrable on  $[0, 1]$ .

4. (a) Let  $f$  be a bounded and monotonically decreasing function on  $[a, b]$ . Prove that  $f$  is Riemann integrable on  $[a, b]$ .

(b) Let  $f(x) = \cos x$ ,  $x \in [0, \pi/2]$ . Prove that  $f$  is Riemann integrable on  $[0, \pi/2]$ .

(c) Is the following statement true? Justify your answer.

If  $f$  is a bounded function, which is not monotone on  $[0, 1]$  (neither monotonically increasing nor monotonically decreasing on  $[0, 1]$ ). Then  $f$  is not Riemann integrable.

5. (a) Let  $f$  be a function continuous on a closed and bounded interval  $[a, b]$ . Prove that  $f$  is Riemann integrable on  $[a, b]$ .

(b) Suppose  $f$  is Riemann integrable on  $[a, b]$  and  $a < c < b$ . Prove that

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx.$$

(c) Let  $f(x) = \begin{cases} x, & x \in [0, 1], \\ x^2, & x \in (1, 2]. \end{cases}$  Evaluate  $\int_1^3 f(x) dx$ .

6. (a) State and prove the Mean-Value Theorem for integrals.

(b) State the Generalized Mean-Value Theorem and prove that the Generalized Mean-Value theorem implies the Mean-Value theorem.

(c) Determine convergence of  $\int_2^3 \frac{1}{3-x} dx$ .