00107

The Open University of Sri Lanka
Department of Mathematics and Computer Science
B.Sc/ B.Ed Degree Programme
Final Examination - 2013/2014
Applied Mathematics– Level 05
APU3244/ APE5244– Graph Theory

DURATION: - THREE HOURS

Date: - 27 – 11 – 2014 **Time: - 1.30 p.m. – 4.30 p.m.**

ANSWER FIVE QUESTIONS ONLY

01. Let G be a simple graph with n vertices and l edges. Let f, v, e be the faces, vertices and edges of the graph G respectively. Define a *degree* $\delta(v)$ of a vertex of G.

State the *Handshaking Lemma.*

(a) Draw each of the following graphs:
  (i) A simple graph with 4 vertices of degrees 1, 2, 2 and 3,
  (ii) A complete graph that is a wheel,
  (iii) A complete bipartite graph that is regular of degree 2,
  (iv) A planar graph that is 2-colorable (f), 2-colorable (e) and 2-colorable (v),

(b) Determine whether each of the following statements is true or false. Justify your answer:
  (i) A complete graph K_n has $\frac{n(n-1)}{2}$ number of edges,
  (ii) If each vertex of a graph has degree of at least 3, then the largest possible number of vertices in such a graph with 30 edges is 20,
  (iii) If M and m are the maximum and minimum degrees of the vertices of a graph G respectively then $m \le \frac{2l}{n} \le M$,
  (iv) A regular graph of degree r has $\frac{nr}{2}$ number of edges.

02. (a) Define the *chromatic number* $\chi(G)$ of a graph G. State the *Brook's theorem*.

Hence, compare the upper bound for the chromatic number of the following graphs:

(i) Peterson graph (ii) W_6

(b) A lecture timetable is to be drawn up for 5 subjects *A*, *B*, *C*, *D* and *E*. Since some students wish to attend several lectures, certain lectures must not coincide. The asterisks in the following table show which pairs of lectures cannot be coincided.

	A	*B*	*C*	*D*	*E*
A	--	*	*	*	--
B	*	--	*	*	*
C	*	*	--	*	--
D	*	*	*	--	*
E	--	*	--	*	--

(i) Draw a graph G whose five vertices correspond to the five subjects with two vertices adjacent whenever the corresponding subjects are to be coincided,

(ii) Find the minimum number of periods needed to schedule all five lectures,

(iii) Write down a possible schedule to the above problem,

(iv) Verify the Brook's theorem.

03. (a) Let G be the complete graph K_4. Draw the line graph $L(K_4)$.

Hence, verify the following statements:

(i) "If G is Hamiltonian, then $L(G)$ is a Hamiltonian",

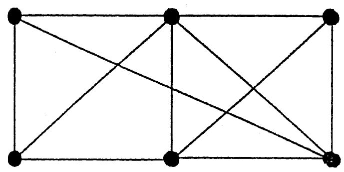
(ii) "If $L(G)$ is both Eulerian and Hamiltonian, then G is an Eulerian".

(b) Draw the total graph $T(K_3)$.

(c) Are $T(K_3)$ and $L(K_4)$ isomorphic? Justify your answer.

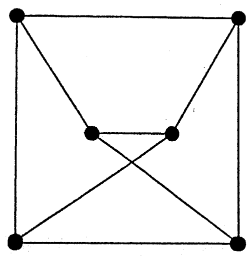
04. Let v be the number of vertices, e be the number of edges and f be the number of faces of a planar graph G. State the *Euler's formula* for a planar graph.

(a) Show that the following graph is planar by drawing it in the plane without crossing edges.

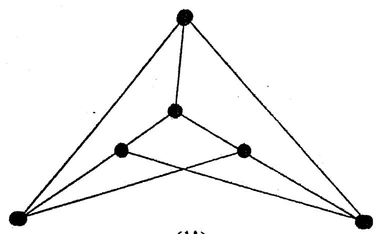


Verify the Euler's formula for the above graph.

(b) Show that the following graphs are non- planar by finding a subgraph homeomorphic to $K_{3,3}$ or K_5:



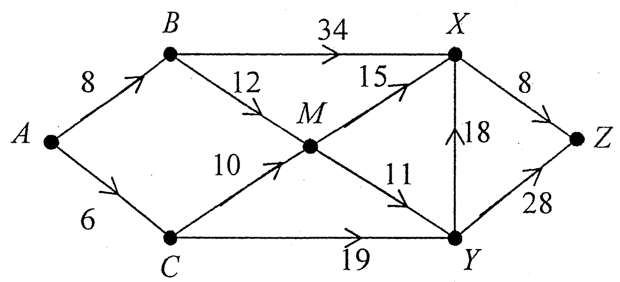
(i)



(ii)

05. State the *Handshaking Dilemma*.

Let the following digraph D represents a construction of a complete house, where A and Z represent the beginning and the completion of the job. Assume that the entire job cannot be completed until each path from A to Z has been traversed.



(i) Obtain the critical path from A to Z,

(ii) Verify the Handshaking Dilemma,

(iii)Is D a tournament? Justify your answer,

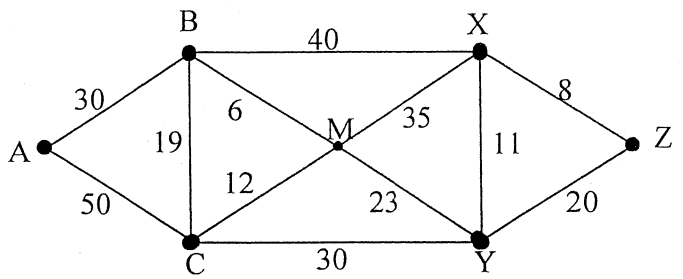
(iv)By using Dijkstra's algorithm, find the shortest path from A to Z.

06. (a) Define a tree.

Let T be a tree and $n \geq 2$. Prove that T contains no cycles and has $n-1$ number of edges.

(b) Draw all non- isomorphic trees of 6 vertices.

(c) Use Prim's Greedy algorithm to find the minimum weighted spanning tree for the following weighted graph, by starting with the edge AB:



Verify the result by using Kruskal's Greedy algorithm.

07. (a) Let G be a complete bipartite graph with one set has n vertices and the other set has m vertices.

(i) Show that G has nm number of edges,

(ii) Prove that each cycle of G has an even length.

(b) Suppose that 3 boys A, B, and C know 4 prospective girls W, X, Y and Z as in the following table.

Boy	Girls known by boy
A	W Y Z
B	X Z
C	X Y

It is given that the boys marry the girls in such a way that each boy marries a girl he knows.

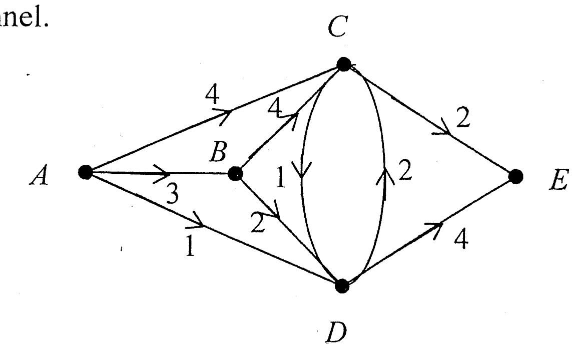
(i) Draw the bipartite graph according to the above relationships,

(ii) Find five different solutions to the marriage problem,

(iii) Check the marriage condition for this problem.

08. State the *maximum flow minimum cut* theorem for a network.

A computer manufacturer wishes to send several computers to a given market. There are various channels through which boxes can be sent, as shown in the following network, with *A* and *E* representing the manufacturer and the market respectively. The number next to each arc refers to the maximum load that can pass through the corresponding channel. The manufacturer wishes to find the maximum number of boxes that can be sent through the network without exceeding the permitted capacity of any channel.



(i) Draw a possible flow for the network,

(ii) Find the maximum flow of the network,

(iii)Find the minimum cut of the network,

(iv) Verify the max-flow min-cut theorem.