THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
APU 3147/ APE 5157 Statistical Inference
FINAL EXAMINATION - 2013/2014

Duration: Two Hours.

Date: 27-11-2014	**Time: 9.30 a.m. to 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.

Answer FOUR questions only.

(1)

Past experience has indicated that the time required for a beginner to become proficient with a particular function of new software product has an approximately a normal distribution.

Time took to become proficient with this particular function of new software product for 15 randomly selected beginners are given bellow.

43.64	39.48	60.78	67.63	68.20
63.70	62.83	52.94	46.92	95.61
60.75	57.10	68.67	41.35	62.66

(i) Estimate the mean time and standard deviation required for a beginner to become proficient with the particular function of new software product.

(ii) Estimate the mean square error of the estimate for the mean time, given by you in the part (i).

(iii) Find the sample size necessary to estimate the average time required for a beginner to become proficient at particular function with the new package within error bound of 5 minutes with 95% confidence.

(iv) Construct 95% confidence interval for mean time required for a beginner to become proficient with the particular function of new software products. Interpret your results.

(2)

(a) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample of size n from a population with mean μ and variance σ^2.

(i) Show that $[\overline{X}]^2$ is a biased estimator for μ^2. Find the bias of this estimator.

(ii) Using part (i) show that $[\overline{X}]^2$ is an unbiased estimator for μ for large enough n.

(b) $\overline{X}_1$ and $S_1{}^2$ are the sample mean and the sample variances from a population with mean μ_1 and $\sigma_1{}^2$. Similarly $\overline{X}_2$ and $S_2{}^2$ are the sample mean and the sample variances from a second independent population with mean μ_2 and variance $\sigma_2{}^2$.The sample sizes are n_1 and n_2 respectively. Suppose both population follows same distribution and $\overline{X}_1 - \overline{X}_2$ is an estimator for $\mu_1 - \mu_2$ Find the mean squared error of $(\overline{X}_1 - \overline{X}_2)$.

(3)

Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \theta \quad ; \quad 0 \leq x \leq \frac{1}{\theta} \ , \ 0 < \theta < 1$$

(i) Find the mean and the variance of the above distribution.

(ii) Derive moment estimator for θ.

(iii) Derive maximum likelihood estimator for θ.

(iv) A sample drawn from the above distribution is given in the following table. Calculate the moment and maximum likelihood estimates for θ based on the estimators derived by you in part (ii) and (iii).

0.92	0.57	1.51	4.75	2.27
1.57	4.12	1.90	0.19	0.82
0.25	3.58	2.51	3.97	3.81
4.45	2.32	1.27	0.72	3.02

(v) Calculate maximum likelihood estimates for mean and the variance of the above distribution based on the given data. State if any properties of maximum likely hood estimators used for above calculation.

(4)

Sample of 15 test tubes were tested for number of times they can be heated on a Bunsen burner. Following table gives the results. Past experience has indicated that number of times they can be heated follows a normal distribution.

1017	1703	1310	1043	1356
1119	1476	951	1241	1221
1188	947	1065	1145	1059
1320	836	1421	999	1284

(i) Construct 95% confidence interval for the variance of the number of times a test tube can be heated on a Bunsen burner. Interpret your results.

(ii) Construct 95% confidence interval for the mean number of times a test tube can be heated on Bunsen burner. Interpret your results.

(iii) The company claims that a randomly selected test tube can be heated on a Bunsen burner 1500 times with a standard deviation of 15. Do you agree with this claim? Justify your answer.

(5)

An experiment is conducted to determine whether the intensive tutoring (covering great deal of material in a fixed amount of time) is more effective than paced tutoring (covering less material in the same amount of time). Two randomly chosen groups are tutored separately and proficiency tests were administrated. Marks (out of 100) for the proficiency tests are given below. Assume that marks of the both methods follows normal distributions with equal variance.

Intensive tutoring	47	52	49	49	48	52	47	53	47	44		
Paced tutoring	55	54	50	54	50	54	52	53	55	56	50	54

Conduct a suitable statistical method to comment on the statement that "Both tutoring methods are equally effective". Write a brief report on your findings with justifications.

(6)

The random variable X denotes the lifetime of a certain type of battery. The probability density function of X is given by

$$f(x,\alpha) = \frac{1}{\alpha}e^{\left(\frac{-x}{\alpha}\right)} \quad ; \alpha > 0, \quad x > 0$$

and the moment generating function of X is given by

$$M_X(t) = (1 - \alpha t)^{-1} \quad ; \quad t < \frac{1}{\alpha}$$

Let $X_1, X_2, \ldots\ldots X_n$ denote lifetimes of n randomly chosen batteries from the above population.

(i) Find the mean and the variance of the above distribution.

(ii) Derive maximum likelihood estimator for mean of the above distribution. Is the estimator derived by you is unbiased for mean? Justify your answer.

(iii) Derive maximum likelihood estimator for variance of the above distribution.

(iv) A sample drawn from the above distribution is given in the following table. Estimate the mean and the variance of the above distribution using the results obtained in part (ii) and part(iii).

0.67	0.17	0.31	1.70
1.07	0.32	1.64	0.19
0.48	0.97	0.04	0.79
0.66	1.48	1.76	4.64
0.25	1.40	1.44	0.99

(v) Estimate the standard error for the estimated mean in part (iv)

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