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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
APU3146/APE5146 - Operations Research
FINAL EXAMINATION 2013/2014
Duration: Two hours

Date: 28.11.2014	**Time: 09.30 a.m- 11.30 a.m**

Answer four questions only

Question 1

(a) Prove that if fixed number M is added to each element of the pay-off matrix, then the optimal strategies remain unchanged while the value of the game increases by M.

(b) There are two competing departmental stores R and C in a city. Both stores have equal reputation and the total number of customers is equally divided between the two. Both the stores plan to run annual discount sales in the last week of December. For this, they want to attract more number of customers by using advertisement through newspaper, radio and television. By seeing the market trend, the store R constructed the following payoff matrix, where the numbers in the matrix indicate a gain or a loss of customers:

Store *C*

Store *R*

	Newspaper	*Radio*	*Television*
Newspaper	40	50	-70
Radio	10	25	-10
Television	100	30	60

(i) Check whether game is strictly determinable? If so find the value of the game.
(ii) Use Dominance property to reduce the given matrix to 2×2.
(iii) Hence or otherwise, find optimal strategies for stores R and C.

Question 2

Suppose people arrive to purchase tickets for a basketball game at the average rate of 4 minutes. It takes an average of 10 seconds to purchase a ticket. If a sports fan arrives 2 minutes before the game starts and if it takes exactly $1\frac{1}{2}$ minutes to reach the correct seat after purchasing a ticket,

(i) can the sports fan expect to be seated for the start of the game? Justify your answer.
(ii) what is the probability that the sports fan will be seated for the start of the game?
(iii) how early must the sports fan arrive in order to be 99% sure of being seated for the start of the game?

Question 3

A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson distribution at an average of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.

(i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?
(ii) If the subscribers will wait and are serviced in turn, what is the expected waiting time?

Question 4

A two-person barber shop has five chairs to accommodate waiting customers. Potential customers, who arrive when all five chairs are full, leave without entering the barber shop. Customers arrive at the average rate of 3.76 per hour and spend an average of 15 minutes in the barber chair.

(i) What is the probability of a customer can get directly into the barber chair upon arrival?
(ii) What is the expected number of customers waiting for a haircut?
(iii) What is the effective arrival rate?
(iv) How much time can a customer expect to spend in the barber shop?
(v) What fraction of potential customers is turned away?

Question 5

(a) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and replenishment rate is finite.

(b) An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the setup cost is Rs.100 per batch and holding cost is Rs.0.01 per unit of item per day.
Using the above formula answer the following questions:

(i) Find the economic lot size for one run,
(ii) Find the time of cycle,
(iii) Determine minimum total cost for one run.

Question 6

(a) Briefly explain the following terms in inventory management:

(i) Setup Cost
(ii) Holding Cost
(iii) Lead Time

(b) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and production rate is infinite.

(c) The demand rate of a particular item is 12,000 units per year. The setup cost per run is Rs.350 and the holding cost is Rs.0.20 per unit per month. If no shortages are allowed, determine

(i) the optimum lot size,

(ii) the number of orders per year,

(iii) the optimum scheduling period,

(iv) the total annual inventory cost,

(v) the total cost per year, if the cost of one unit is Rs.10.

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \frac{1}{n!}\rho^n P_0 & ;1 \le n \le C \\ \frac{1}{C^{n-C}C!}\rho^n P_0 & ;n \ge C \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2}$$

$$E(n) = E(m) + \frac{\lambda}{\mu}$$

$$E(w) = \frac{1}{\lambda}E(m)$$

$$E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;0 \le n \le C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;C \le n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \left\{1-\left(\frac{\lambda}{C\mu}\right)^{N-C+1}\right\}\frac{C\mu}{C\mu-1}\right]^{-1} & ;\frac{\lambda}{C\mu} \ne 1 \\ \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C (N-C+1)\right]^{-1} & ;\frac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o(C\rho)^C\rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1}-(1-\rho)(N-C+1)\rho^{N-C}\right]$$

$$E(n) = E(m) + C - P_0\sum_{n=0}^{C-1}\frac{(C-n)(\rho C)^n}{n!}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1 - P_N)$$

$$E(w) = E(v) - \frac{1}{\mu}$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \ 0 \le n < R \\ \binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ; \ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right]$$

$$E(m) = \sum_{n=R}^{K}(n - R)P_n$$

$$E(v) = \frac{E(n)}{\lambda[K - E(n)]}$$

$$E(w) = \frac{E(m)}{\lambda[K - E(n)]}$$

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