The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 05
Final Examination 2013/2014
Applied Mathematics
APU 3145/ APE5145 – Newtonian Mechanics II

Duration :- Two Hours

Date :-24.11.2014 **Time:-01.30 p.m. -03.30p.m.**

Answer Four Questions Only.

1. (a) State D' Alembert's principle.

   (b) A light straight rod OAB such that $OA = a$, $OB = b$ can turn freely in a vertical plane about a smooth fixed hinge at O. Two heavy particles of masses m and m' are attached to the rod at A and B respectively and oscillate with it. Using D' Alembert's principle, show that the equation of motion is given by $(ma^2 + m'b^2)\ddot{\theta} + (ma - m'b)g\sin\theta = 0$

2. (a) In the usual notation, show that, in spherical polar coordinates, the velocity and acceleration of a particle are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\underline{k}$ and

   $$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$$

   respectively.

   (b) A particle is projected horizontally with velocity u along the interior surface of a smooth hemisphere whose axis is vertical and whose vertex is downwards. The radius through the point of projection makes angle β with the downward vertical. If the particle just ascend to the rim of the hemisphere show that $u = \sqrt{2ag\sec\beta}$, where a is the radius of the hemisphere.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

   (b) A projectile located at a point of latitude λ is projected with speed v_0 in a southward direction at an angle α to the horizontal. Find the position of the projectile after time t. Prove that after time t, the projectile will be deflected towards the east of the original vertical plane of motion by the amount $\frac{1}{3}\omega g\cos\lambda t^3 - \omega v_0\sin(\alpha+\lambda)t^2$.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative holonomic system with n degrees of freedom are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0, \quad j=1,2\ldots,n.$$

(b) A uniform rod AB of length 2a is suspended from a fixed point O by a string OC of length $5a/6$, attached to a point C of the rod such that $AC=2a/3$. The system moves in a vertical plane with the string taut. Let the inclinations ofOC and AB to the vertical at time t be q_1 and q_2 respectively. Show that the kinetic energy T and the potential energy V are given by

$$2T=\frac{1}{36}ma^2\left(25\dot{q}_1^2+16\dot{q}_2^2+20\dot{q}_1\dot{q}_2\right) \text{ and } V=-mga\left(\frac{5}{6}\cos q_1+\frac{1}{3}\cos q_2\right)+Const.$$

Hence write down the Lagrangian of the system and thus obtain the equations of motion.

05. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) The principal moments of inertia of a body at the centre of mass are A, $3A$, $6A$. The body is initially rotated with an angular velocity having components about the principal axes $3n$, $2n$, n respectively. In the subsequent motion under no forces, if ω_1, ω_2, ω_3 denote the angular velocities about the principal axes at time t, show that

$$\omega_1=3\omega_3=\frac{9n}{\sqrt{5}}\operatorname{sech} u \quad \text{and} \quad \omega_2=3n\tanh u\text{, where } u=3nt+\frac{1}{2}\ln 5.$$

06.(i) (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i}=\dot{q}_i$, $\frac{\partial H}{\partial q_i}=-\dot{p}_i$.

(b) Using Hamilton's equations find the equations of motion of a projectile in three dimensional space.

(ii) (a) Define a canonical transformation.

(b) Show that the transformation $Q=\log\left(\frac{1}{q}\sin p\right)$, $P=q\cot p$ is canonical.