THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme

APPLIED MATHEMATICS-LEVEL 05

AMU3186/AME5186- Quantum Mechanics

FINAL EXAMINATION- 2013/2014

Duration: Two Hours.

Date: 02.12.2014	**Time: 1.30 p.m. to 3.30 p.m.**

Answer **FOUR** Questions only.

(1) An X-ray photon of wave length $\lambda = 10^{-10}\, m$ is incident on a stationary electron, where $\lambda_c = \frac{h}{mc}$ is the Compton wave length, m being the mass of the electron and c, the speed of light.

(i) Show that for Compton scattering the wavelength shift

$\delta\lambda = \lambda' - \lambda = 2\lambda_C \sin^2\frac{\theta}{2}$, where λ is the wave length of the incident X-ray and λ' is the wave length of X-ray scattered through an angle θ.

(ii) Calculate the Compton shift.

(iii) Calculate the kinetic energy of the recoiling electron, if

$m = 9.108 \times 10^{-31}\, kg,\quad c = 3 \times 10^8\, ms^{-1},\quad h = 6.625 \times 10^{-34}\, Js$ and $\theta = 30^\circ$.

(2) (i) Define the commutator, usually denoted by $[A, B]$, of two operators $\hat{A}$ and $\hat{B}$.

(ii) If $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ are operators, show that

(a) $[\hat{A},\hat{B}] = -[\hat{B},\hat{A}]$

(b) $[\hat{A},\hat{B}\hat{C}] = [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}]$

(c) $[\hat{A},\hat{B}+\hat{C}+\hat{D}] = [\hat{A},\hat{B}] + [\hat{A},\hat{C}] + [\hat{A},\hat{D}]$

(d) $[\hat{A}+\hat{B},\hat{C}+\hat{D}] = [\hat{A},\hat{C}] + [\hat{A},\hat{D}] + [\hat{B},\hat{C}] + [\hat{B},\hat{D}]$

(iii) Prove that, in a usual notation,

(a) $\left[\hat{x},\hat{P}_x\right] = i\hbar$

(b) $\left[\hat{P}_x,\hat{x}^2\right] = -2i\hbar x$

(3) The parity operator $\hat{\Pi}$ is defined by the operation $\hat{\Pi}\Psi(x) = \Psi(-x)$.

(i) Show that

(a) It is a linear operator.

(b) It is a Hermitian operator.

(ii) Find the eigen values of this operator $\hat{\Pi}$.

(iii) If the potential energy V is an even function, then show that $\hat{\Pi}$ commutes with the Hamiltanian $\hat{H} = \frac{\hat{P}^2}{2m} + V(x)$.

(4) (a) If $\hat{A}$ is an operator corresponding to a quantum observable and $\langle \hat{A} \rangle$ is the corresponding expectation value, Show that

$$\frac{d\langle \hat{A} \rangle}{dt} = \langle \frac{\partial \hat{A}}{\partial t} \rangle + \frac{1}{i\hbar} \langle [\hat{A}, \hat{H}] \rangle.$$

(b) A particle of mass m and energy E moves in the positive x direction under square hill potential defined by

$$V = \begin{cases} 0 & ;\ x < 0 \\ V_0 > 0 & ;\ 0 < x < a \\ 0 & ;\ a < x \end{cases}$$

Find the wave function $u(x)$ for each of the regions given above, for cases $E < V_0$ and $E > V_0$.

(5) One dimensional wave function $\psi(x, t)$ is given by

$$\psi(x,t) = B\ Sin\left(\frac{\pi x}{2a}\right) e^{-i\alpha t} \quad ; 0 \le x \le a$$ where α is a constant.

(a) If ψ is normalized, find B.

(b) Calculate the mean vlaues of x, x^2 and $\hat{P}_x$ with respect to $\psi(x)$.

(c) Calculate (Δx).

(6) The angular momentum of a particle is defined as a vector $\underline{L}$, such that $\underline{L} = \underline{r} \times \underline{p}$,

where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle with respect to a fixed origin O.

(a) Write down the Cartesian components $\hat{L}_x, \hat{L}_y, \hat{L}_z$ of the angular momentum operator.

(b) Hence obtain the angular momentum operator in spherical polar coordinates (r, θ, ϕ).
Use $\hat{\theta} = \cos\theta\cos\phi\ \underline{i} + \cos\theta\sin\phi\ \underline{j} - \sin\theta\ \underline{k}$ and $\hat{\phi} = -\sin\phi\ \underline{i} + \cos\phi\ \underline{j}$

(c) Show that $\left[\hat{L}_y, \hat{L}_z\right] = i\hbar\hat{L}_x$ and $\left[\hat{L}^2, \hat{L}_x\right] = 0$.

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