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The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Final Examination-2013/2014

Applied Mathematics – Level 05

AMU3185 /AME5185 – Electro-Magnetic Theory & Special Relativity

Duration: - Two Hours

Date: 26th November 2014 **Time: 1.30 p.m. - 3.30 p.m.**

Answer Four Questions Only.

You may, wherever necessary, assume that $4\pi\varepsilon = \frac{10^{-9}}{9} Fm^{-1}$.

1) (i) State Coulomb's law, used in electrostatic theory.

(ii) Two identical pith balls, each of mass m, and carrying a charge Q are suspended from the same point by two weightless strings of length l. If α ,the inclination angle of each thread to the vertical is very small at the equilibrium, show that

$$\alpha = \sqrt[3]{\frac{Q^2}{16\pi\varepsilon_0 mgl^2}}$$

where ε_0 permittivity of medium.

(iii) Three point charges $Q_1 = 1mC$, $Q_2 = 2mC$, and $Q_3 = -3mC$ are located at $(0,0,4),(-2,6,1)$ and $(3,-4,-8)$ respectively. Calculate the force on Q_1.

2) (i)Define following terms, used in electromagnetic theory.

a) Electric field

b) Neutral Point

(ii) Prove the following identities:

a) $\nabla.\nabla\times\underline{A} = 0$

b) $\nabla\times\nabla V = 0$

where $\underline{A}$ is any vector potential function and V is any scalar potential function.

(iii) The gradient of a scalar potential function ϕ is given by,

$$\nabla\phi = 3\left[\left(x^2 - yz\right)\underline{i} + \left(y^2 - xz\right)\underline{j} + (z^2 - xy)\underline{k}\right].$$

If the point (1,1,1) is at zero-potential, determine the potential function ϕ.

3) (i) State Gauss law, used in electromagnetic theory.

(ii) Suppose a point charge Q is located at the origin. Determine the electric flux intensity due to this charge at a general point $P(r,\theta,\varphi)$.

(iii) A charge distribution with spherical symmetry has density

$$\rho_v = \begin{cases} \dfrac{\rho_0 r}{R}, & 0 \le r \le R \\ 0, & r > R \end{cases}$$

Determine the electric field, E for all regions.

4) (i) Write down Poisson's and Laplace's equations in Cartesian, cylindrical and spherical coordinates.

(ii) Show that following potential satisfies Laplace's equation

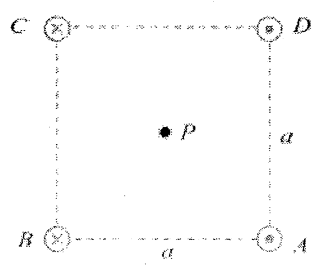
$V = 15x^2yz - 5y^3z$.

(iii) In cylindrical polar coordinates, for a charge distribution, its density is given by $\rho_v = \dfrac{10}{\rho} \times 10^{-12} Cm^{-3}$. If electric potential, $V = 0$ at $\rho = 1m$ and $V = 100\text{V}$ at $\rho = 4m$, find the V at $\rho = 3m$.

5) (i) State Biot-Savart law, used in electromagnetic theory.

(ii) Four infinitely long parallel wires carrying equal current I are arranged in such a way that when looking at the cross section, they are at the corners of a square, as shown in the figure. Currents in A and D point out of the page and those at B and C into the page. What is the magnetic field at the centre of the square?

(Hint: The magnitude of the magnetic field a distance r from an infinite wire is $B = \dfrac{\mu_0 I}{2\pi r}$)



(iii) In a certain conducting region magnetic field intensity,

$$H = yz(x^2 + y^2)\underline{a}_x - y^2xz\underline{a}_y + 4x^2y^2\underline{a}_z \; Am^{-1}.$$

Show that $\nabla.B = 0$.

where, B is magnetic flux intensity.

6) (i) State the meaning of Galilean Transformations.
(ii) Derive the Lorentz transformation equations.
(iii) With the usual notation verify that the above equations can be expressed in the form

$$x' = x\cosh\alpha - ct\sinh\alpha$$
$$y' = y$$
$$z' = z$$
$$ct' = ct\cosh\alpha - x\sinh\alpha.$$

where $\tanh\alpha = \frac{v}{c}$.

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