The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2009/2010
ECX6234 – Digital Signal Processing

Time: 0930 – 1230 hrs. Date: 2010-03 -13

Answer any FIVE questions

Q1.

(a) The signal $x(t)$ has to be transformed into the sequence $x[n]$. How would you do this?

(b) Calculate the frequency components present in following sequences:

(i) $x[n]=[1,-1,3,-1]$.

(ii) $x[n]=[1,0,1,0,1,0,1,0]$.

(c) Calculate the Discrete Time Fourier Transform (DTFT) of $a^n u[n]$ if $a<1$.

(d) Fig.1 shows two sequences $x[n]$ and $h[n]$.

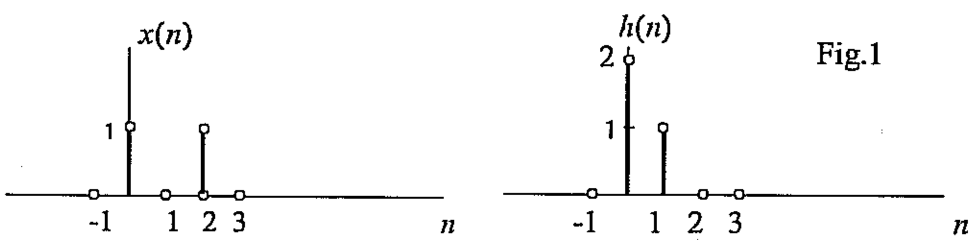

Fig.1

Find the convolution $y[n]=x[n]*h[n]$.

(e) A Linear Time Invariant (LTI) system has an impulse response given by $h[n]=0.2^n u[n]$.

(i) Define a LTI system.

(ii) Determine the output sequence $y[n]$ if

(1) $x[n]=u[n]$.

(2) $x[n]=1.6u[5-n]$.

Q2.

(a) (i) Define z-transform $X(z)$ of a signal $x[n]$.

(ii) Using the definition given above find the z-transforms of

(1) $x[n-n_0]$

(2) $z_0^n x[n]$

in terms of $X(z)$, the z-transform of $x[n]$. z_0 is a constant.

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(iii) Find the z-transforms for the following:

(1) $x[n] = 0.3^n u[-n-1]$.

(2) $x[n] = \delta[n+1] + \delta[n+2] + \delta[n+3]$

(b) A causal system is characterized by the difference equation

$$y[n] = 0.5y[n-1 + x[n]$$

(i) Find the transfer function $H(z) = \frac{Y(z)}{X(z)}$

(ii) Find the impulse response $h[n]$.

(iii) Now $h[n]$ is truncated into $h'[n]$ as follows:

$h'[n] = h[n]$ for $0 \le n \le 8$

$= 0$ elsewhere

(1) Find the zero's and poles of the new system function $H'(z)$.

(2) Sketch the pole-zero plot.

Some important Z-transforms

Function	z-transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{z}{z-1}$	$\|z\|>1$
$a^n u[n]$	$\frac{z}{z-a}$	$\|z\|>a$
$-a^n u[-n-1]$	$\frac{z}{z-a}$	$\|z\|<a$
$nx[n]$	$X\left(\frac{1}{z}\right)$	$R' = \frac{1}{R}$
$x[-n]$	$-z\frac{dX(z)}{dz}$	$R' = R$
$e^{j\Omega_0 n}x[n]$	$X(e^{j\Omega_0 n}z)$	R'=R

$X(z)$ is the z-transform of $x[n]$. R is the ROC of $X(z)$

Q3.

A Finite impulse response filter (*FIR*) has an impulse response $h(n)$ and the frequency response $H(\omega)$.

(a) Write the relationship between $h(n)$ and $H(\omega)$. [Hint: understand the difference between FIR- and IIR (Infinite Impulse Response) filters].

(b) Using the expression in (a) write an expression for the duration of the impulse response.

(c) If $h(n) = h(-n)$ show that $H(\omega)$ has a zero phase.

(d) Using the fact that $\cos n\theta$ can be expressed as a sum of powers of $\cos\theta$, show that $H(\omega)$ can be written as

$$H(\omega) = \sum_{k=0}^{M} a_k \cos^k \omega$$

(e)

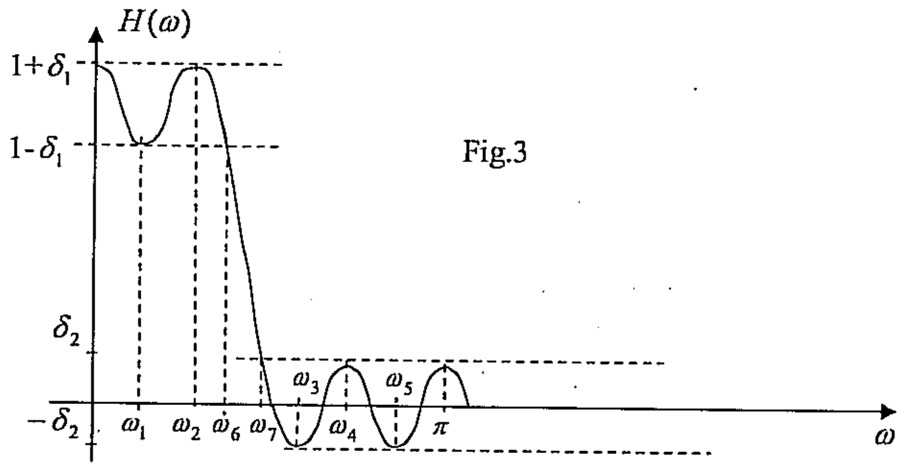

Fig.3

Equi-ripple approximation for a low-pass filter is given in Fig.3.

(i) Give the values of pass band frequency (ω_p) and stop band frequency (ω_s) for the filter given in Fig.3.

(ii) What should be our design algorithm? (What are we going to achieve in the design?)

(f) Suppose we want to design an equiripple filter having the transfer function given in (d).

(i) Find $\dfrac{dH}{d\omega}$.

(ii) How many local minima- and maxima can $H(\omega)$ have at most?

(g) (i) Find a value for M (refer (d))

(ii) For this filter show that sufficient number of equations can be written, in order to find the values of $a_k, \omega_1, \omega_2, \ldots \omega_5$.

Q4.

(a) Write an expression for Discrete Fourier Transform (*DFT*) of $x[n]$.

(b) Two sequences $x[n]$ and $y[n]$ are related to each other as follows:

$$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

If the *DFT*'s of $x[n]$ and $y[n]$ are $X(k)$ and $Y(k)$ respectively, find the relationship between $X(k)$ and $Y(k)$.

(c) (i) Write an expression for Inverse Discrete Fourier Transform (*IDFT*).

(ii) *DFT* of a finite sequence $x[n]$ is given by $X(k)$, $k = 0,\ldots N\text{-}1$

Show that $x_p[n] = x[n]$ represents a periodic sequence.

Q5.

(a) A signal flow graph of a digital system is given below:

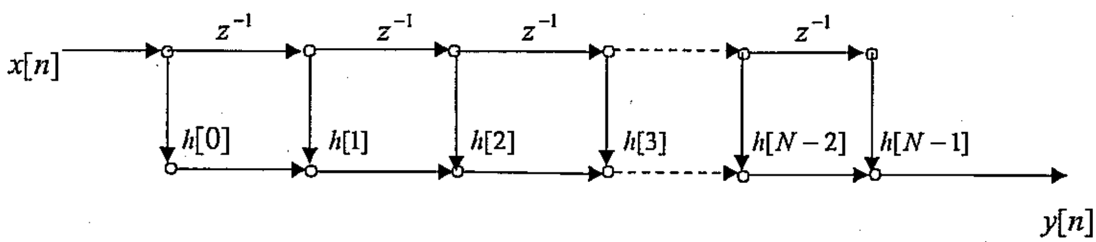

Fig.5(a)

(i) Write an expression for $y[n.]$

(ii) Find $y[n]$, if

(1) $x[n] = \delta[n]$

(2) $x[n] = u[n]$

(b)

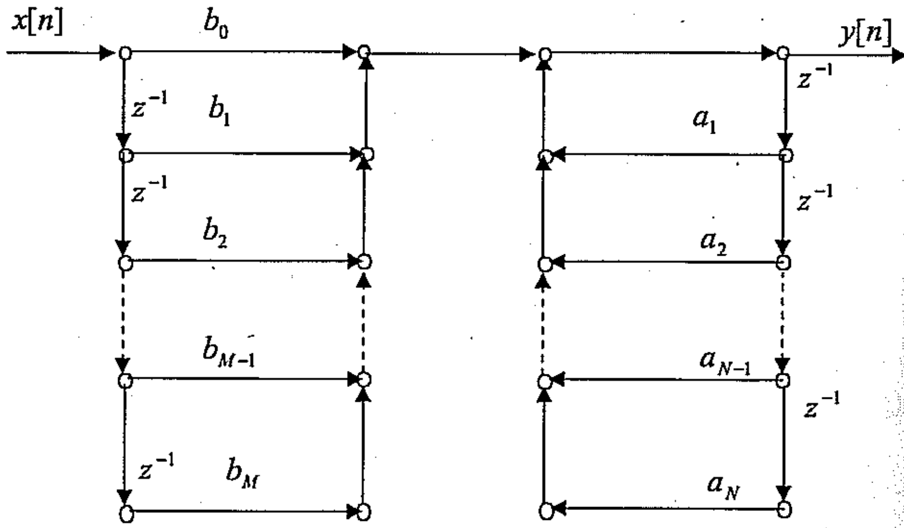

Fig.5(b)

For the signal flow graph given in Fig.5(b) show that the transfer function is given by

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

(c) The transfer function $\frac{Y(z)}{X(z)}$ of a digital system is given by

$$\frac{Y(z)}{X(z)} = \frac{1}{a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}}$$

Show that the impulse response $h[n]$ satisfies the equation

$$\delta[n] = \sum_{i=1}^{N} a_i h[n-i]$$

Q6.

(a) Why cannot we realize an ideal low-pass filter in practice?

(b) How can we overcome the above difficulties using a window function?

(c) Suppose we use a Blackman window for this purpose. It has a transition region width ($\Delta\omega$) of $\frac{12\pi}{N}$, where N is the filter length. The digital frequencies corresponding to pass band (ω_p) and stop band (ω_s) are 1.5 *radians* and 2 *radians* respectively.

$h_d[n]$ = impulse response of an ideal low-pass filter.
$h_w[n]$ = impulse response of the resulting filter after windowing.
$h[n]$ = impulse response of the final, practical low-pass filter filter.
$w[n]$ = window function.

(i) Write equations to show the relationship between

(1) $h[n]$ and $h_w[n]$.
(2) $h_d[n]$ and $h_w[n]$.

(ii) In the equation relating $h[n]$ and $h_w[n]$ there is an unknown parameter.
(1) What does this parameter physically mean?
(2) Calculate the value of the parameter.

(iii) Find the length of the filter.

(d) What criteria would you adopt when selecting a suitable window type in the realization of a practical low-pass filter?

Q7.

(a) What is meant by

(i) up sampling
(ii) down sampling

of a digital signal?

(b) (i) Show that the sequence $\delta_D[n] = \frac{1}{D}\sum_{k=0}^{D-1} e^{-j2\pi kn/D}$ can be used to sample a given sequence $x[n]$. (Note that $\lim_{x\to 1}\left(\frac{1-x^a}{1-x}\right) = a$)

(ii) If $x[n] = [1,1,...,1]$, and $D = 4$, give the resulting sequence after sampling.

(iii) A sequence $s[n]$ when sampled using $\delta_D[n]$, is converted into a new sequence $y[n]$.

(1) Find $Y(z)$, the z-transform of $y[n]$.

(2) Show that $Y(\omega) = \frac{1}{D}\sum_{k=0}^{D-1} S(\omega + 2\pi k/D)$.

where $S(\omega)$ and $Y(\omega)$ are the Discrete Time Fourier Transforms of $s[n]$ and $y[n]$ respectively.

(c) It is necessary to design a low pass filter using *multistage decimation technique*. Specifications of the filter are given below:

pass band $f_p = 500\ Hz$.

stop band $f_s = 550\ Hz$.

sampling frequency $f_x = 84\ kHz$

For each stage select / calculate

(i) a suitable sampling frequency f_i.
(ii) a suitable stop band frequency (ω_s).
(iii) a suitable pass band frequency (ω_p).

Q8.
(a)

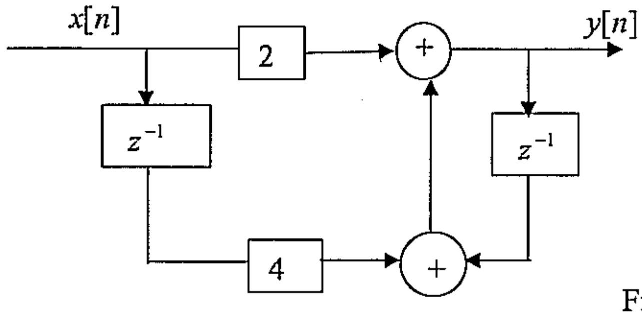

Fig.8

Consider the block diagram realization given in Fig.8.

Express the system in the state-space form.

i.e. Find matrices A, B, C & D such that

$$s[n+1] = A\,s[n] + B\,x[n]$$
$$y[n] \quad = C\,s[n] + D\,x[n]$$

where $s[n]$ is the state vector.

(b) State-space equations for a certain system is given by

$$s[n+1] = 0.8\,s[n] + x[n]$$
$$y[n] \quad = 3\,s[n]$$

(i) Find the transfer function $H(z)$.

(ii) Find the impulse response $h[n]$.

(iii) Is the system BIBO stable? Justify your answer.