The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme- Level 04
Final Examination- 2013/2014
Applied Mathematics
AMU 2181/AME 4181-Mathematical Modelling I

Duration:- Two Hours

Date:-23.06.2014 Time:-9.30a.m.-11.30a.m.

Answer Four Questions Only

(1) The input and output per production run of two possible blending processes of an oil refinery are as follows:

	Input		Output	
Process	Crude A	Crude B	Gasoline C	Gasoline D
1	5	3	5	8
2	4	5	4	4

The maximum amount available of Crude A and Crude B are 200 units and 150 units respectively. Market requirements show that at least 100 units of Gasoline C and 80 units of Gasoline D must be produced. The profits per production run from process 1 and process 2 are Rs. 3 and Rs. 4 respectively. It is required to decide on the optimal mix of the two blending processes that leads to the maximum profit. Formulate the problem as a linear programming problem and solve it using graphical method.

(2) A manufacturer of leather belts makes three types of belts A, B and C which are processed on three machines M_1, M_2 and M_3. Processing time (in hours) of each belt on each machine, daily capacity of each machine and profit margin (in rupees) of each belt are given below:

Belt / Machine	A	B	C	Total
M_1	2	3	-	8
M_2	-	2	5	10
M_3	3	2	4	15
Profit (Rs) Margin	3	5	4	

Assuming that all belts produced will be sold, formulate a mathematical model to maximize the profit. Hence, solve the model and write the optimal solution.

(3) Solve the following problem using big M method:

Minimize $Z=5x_1+3x_2$

Subject to

$$x_1+2x_2 \leq 6$$
$$x_1+x_2 = 5$$
$$5x_1+2x_2 \geq 10$$
$$x_1,\ x_2 \geq 0$$

Verify the solution using the graphical approach.

(4) Consider the following linear programming problem:

Minimize $Z = 24x_1 + 30x_2$

Subject to

$$2x_1 + 3x_2 \geq 10$$
$$4x_1 + 9x_2 \geq 15$$
$$3x_1 + 3x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

Formulate the dual of the above problem. Solve the dual problem and hence, write the optimal solution of the primal problem.

(5) Consider the following linear programming problem:

Maximize $Z = x_1 + 4x_2$

Subject to

$$x_1 + 6x_2 \leq 24$$
$$x_1 \leq 6$$
$$x_1, x_2 \geq 0,$$

where x_1 and x_2 are quantities of two products produced by a company, and the constraints represents available raw materials and machine time.

(a) Find the optimum solution for this linear programming problem.
(b) Find the range over which the objective function can vary for x_1 and the current solution still remains optimal.
(c) Give an interval for the materials resource over which the current solution remains optimal.

(6) Consider the following problem:

$$\text{Maximize } Z = 45x_1 + 100x_2 + 30x_3 + 50x_4$$

Subject to

$$7x_1 + 10x_2 + 4x_3 + 9x_4 \le 1200$$

$$3x_1 + 40x_2 + x_3 + x_4 \le 800$$

$$x_1, x_2, x_3, x_4 \ge 0$$

The optimal simplex tableau for the above problem is given below:

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	requirement
x_3	5/3	0	1	7/3	4/15	-1/15	800/3
x_2	1/30	1	0	-1/30	-1/150	2/75	40/3
Z	-25/3	0	0	-50/3	-22/3	-2/3	

If a new variable x_7 is added to this problem with a column $\begin{pmatrix}10\\10\end{pmatrix}$ and corresponding cost coefficient $(c_7) = 120$, find the change in the optimal solution.