

The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme-Level-05

Department of Mathematics and Computer Science

Final Examination- 2013/2014

Computer Science

CPU3140- Mathematics for Computing

Duration: Two Hours



Date :07.06.2014

Time: 9.30am-11.30am

Answer Four Questions Only

01. (i) If A and B are any two sets, write the following using set notation.

(a) Distributive Law

(b) Complement Law

(c) Identity Law

(ii) Using the above laws, prove the identity  $(A \cup B) \cap (A \cup B^c) = A$

(iii) If  $A_1, A_2, \dots, A_n$  are sets such that  $A_i \cap A_j = \emptyset$  for  $1 \leq i, j \leq n$  and  $i \neq j$

Prove that  $|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|$

(iv) Find the number of mathematics students at a college taking at least one of the

languages, French, German and Russian. Following data is given. 65 study French, 45 study

German, 42 study Russian, 20 study French and German, 25 study French and Russian, 15

study German and Russian and 8 study all three languages.

02. (i) Find the cardinal number of each of the sets given below.

(a) { Monday, ..... Sunday }

(b) {  $x$  :  $x$  is a letter in the word { "BASEBALL" } }

(c) {  $x$  :  $x^2=9, 2x=8$  }

(d) The power set of  $A = \{1, 5, 7, 11\}$

(ii) Let  $f, g, h$  be functions from  $\mathbb{N} \rightarrow \mathbb{N}$  where  $\mathbb{N}$  is the set of natural numbers so that

$$F(n)=n+1, g(n)=2n$$

$$h(n)=0; \text{ when } n \text{ is even and}$$

$$h(n)=1; \text{ when } n \text{ is odd.}$$

Determine the following composite functions.

$$(a) f \circ f, (b) f \circ g \quad (c) g \circ f$$

$$(d) g \circ h; \text{ for even and odd } n$$

$$(e) (f \circ g) \circ h; \text{ for even and odd } n$$

03. (i) Find the truth set of each propositional function  $p(x)$  defined on the set  $\mathbb{N}$  of positive integers.

$$(a) \text{ Let } p(x) \text{ be } "x+2 > 7"$$

$$(b) \text{ Let } p(x) \text{ be } "x+5 < 3"$$

$$(c) \text{ Let } p(x) \text{ be } x+5 > 1"$$

(ii) Write the following statements in symbolic forms.

(a) A program is readable only if it is well structured.

(b) Unless he studies he will fail the examination.

(c) If I am not in good mood or I am not busy then I will go for a movie.

(d) If you know OOP or Oracle, then you will get a job.

04. (i) If  $p$  and  $q$  are two propositions, give truth tables for Conjunction, Disjunction, Conditional implication and Bi-conditional.

(ii) Using part (i) construct the truth tables and verify whether each of the following is a tautology, a contradiction or a contingency.

$$(a) (p \wedge q) \wedge \sim(p \vee q)$$

$$(b) (p \vee \sim q) \rightarrow (p \wedge q)$$

$$(c) (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$$

05. (i) Find  $x, y, z$  and  $t$  where

$$3 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

(ii) Let  $f(x) = 2x^3 - 4x + 5$ . Find,  $f(A)$  where  $A$  is a matrix given by

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & -3 & 6 \\ 1 & 1 & 7 \end{bmatrix}$$

(iii) Find the determinant and the inverse of the matrix given by  $B$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

06. (a) Classify the following recurrence relation

(i)  $a_n = a_{n-1} + a_{n-3}^2$

(ii)  $a_n = a_{n-1} + a_{n-2}$

(iii)  $a_n = 5a_{n-1} - 4a_{n-2} + n^2$

(iv)  $a_n = na_{n-1} + 3a_{n-2}$

(v)  $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$

(b) A recurrence relation is given by

$$t_n = 3t_{n-1} + 4t_{n-2} \text{ with } t_n = 0 \text{ when } n=0 \text{ and } t_n = 5 \text{ when } n=1$$

(i) What is the order of the recurrence relation?

(ii) Find the general solution of the given recurrence relation.

(iii) Hence, find the particular solution.

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