The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2013/2014
Applied Mathematics
AMU3183/AME5183 — Numerical Methods II

Duration: Two Hours

Date: 09. 06. 2014 **Time: 01.00 p.m. – 03.00 p.m.**

Answer Four Questions Only.

1. (a) Prove that

   (i) $E = \Delta + 1,$ (ii) $E = (1-\nabla)^{-1},$

   where Δ, ∇ and E are the forward difference, the backward difference and the shift operators respectively

   (b) Derive the Gregory- Newton backward interpolation formula. Hence, interpolate $f(42)$ corresponding to the data points (20, 354), (25, 332), (30, 291), (35, 260) , (40, 231) and (45, 204).

2. (a) Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3}dx$, using Simpson's One third rule, by dividing the interval into 4 equal parts.

   (b) Find the Lagrange polynomial (f) passing through the points (-1, -1), (-2, -9), (2, 11) (4, 69) and determine $f(0)$.

3. (a) Derive formula for the Euler's method to solve $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

   (b) Solve $\frac{dy}{dx} = 1 + y^2$ with the initial condition $y(0) = 0$, using Euler's method at $x = 0.5$ correct to 4 decimal places, with $h = 0.1$

42

4. (a) Derive formula for the modified Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 0$ using the modified Euler's method, at $x = 0.6$, $x = 0.8$ and $x = 1.0$.

5. (a) Using the Taylor series method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, with the initial condition $y(0) = 0$ at $x = 0.1$ and $x = 0.2$.

(b) Using the Taylor series method, solve $\frac{d^2y}{dx^2} + xy = 0$, with the initial condition $y(0) = 1$ and $y'(0) = 0.5$ at $x = 0.1$ and $x = 0.2$.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using fourth order Runge-Kutta method, solve $\frac{dy}{dx} = xy + y^2$ with the initial condition $y(0) = 1$ at $x = 0.1$ and $x = 0.2$.