The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2013/2014
Applied Mathematics
APU3240/APE5240 — Numerical Methods

Duration: Three Hours

Date: 09. 06. 2014 **Time: 01.00 p.m. – 04.00 p.m.**

Answer Five Questions Only.

1. (a) Briefly explain simple iterative method.

   (b) Let $x=\xi$ be a root of $f(x)=0$ such that $\xi \in I$ where I is an interval. Let $\phi(x)-x \equiv f(x)$ such that $\phi(x)$ and $\phi'(x)$ be continuous in I. Prove that if $|\phi'(x)|<1$ for all $x \in I$ the sequence $x_1, x_2, \ldots x_n$ defined by $\phi(x_n)=x_n$ converges to the root ξ provided that initial approximation $x_0 \in I$.

   (c) Using simple iterative method, find the root of the equation $\sin x - 3x + 1 = 0$ lying in the interval $[0, 0.5]$ and correct to four decimal places.

2. (a) Prove that

   (i) $E=\Delta+1$, (ii) $E=(1-\nabla)^{-1}$, (iii) $\delta = E^{1/2}-E^{-1/2}$,

   (iv) $\mu=\frac{1}{2}\left(E^{1/2}-E^{-1/2}\right)$, (v) $\mu\delta=\frac{\Delta+\nabla}{2}$,

   where Δ, ∇, δ, μ and E are the forward difference, the backward difference, the central difference, the average and the shift operators respectively.

   (b) Derive the Gregory- Newton backward interpolation formula. Hence, interpolate $f(42)$ corresponding to the data points (20, 354), (25, 332), (30, 291), (35, 260) , (40, 231) and (45, 204).

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3. (a) Show that cubic spline interpolation polynomial that passes through the points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ is given by

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right).$$

(b) Find the cubic splines interpolation polynomial that passes through the points $(1, 1)$, $(2, 2)$, $(3, 5)$ and $(4, 11)$.

4. (a) Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3}dx$, using Simpson's One third rule, by dividing the interval into 4 equal parts.

(b) Find the Lagrange polynomial (f) passing through the points (-1, -1), (-2, -9), (2, 11) (4, 69) and determine $f(0)$.

5. (a) (i) Derive formula for the Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = 1 + xy$ with the initial condition $y(0) = 1$.

(b) (i) Derive formula for the modified Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Solve $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 0$ using the modified Euler's method, at $x = 0.6$, $x = 0.8$ and $x = 1.0$.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using fourth order Runge-Kutta method, solve $\frac{dy}{dx} = xy + y^2$ with the initial condition $y(0) = 1$ at $x = 0.1$ and $x = 0.2$.

(c) Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, with the initial condition $y(0) = 1$, $y'(0) = 5$, using fourth order Runge-Kutta method at $x = 0.1$. ,

7. (a) State Milne's Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = x(x^2 + y^2)e^{-x}$, $y(0) = 1$, by Taylor series method for $x = 0.1, 0.2, 0.3$ and hence find $y(0.4)$ by Milne's Predictor – Corrector Method.

8. (a) State Adam-Bashforth Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$, by Taylor series method for $x = 0.1, 0.2, 0.3$ and hence find $y(0.4)$ by Adam-Bashforth Predictor – Corrector Method.