The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2013/2014
Applied Mathematics-Level 05
AMU3182/AME5182– Mathematical Methods I

Duration: - Two hours

Date:-23.06.2014 **Time: - 1.00p.m. – 3.00p.m.**

Answer 4 questions only

(1) (a) Find the general solution of the following system of simultaneous differential equations:

$$\dot{x}_1 = 7x_1 - 3x_3$$
$$\dot{x}_2 = -9x_1 - 2x_2 + 3x_3$$
$$\dot{x}_3 = 18x_1 - 8x_3 \quad .$$

(2)

(b) (i) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0.$$

(ii) Find the solution of part (i) for which $y = 3$ and $\frac{dy}{dx} = 0$, when $x = 1$.

(2) (a) Solve each of the following partial differential equations:

(i) $x\frac{\partial u}{\partial x} + \frac{1}{x(1+y^2)}u + 2yxe^{\frac{1}{x(1+y^2)}} = 0; \quad u = u(x,y)$

(ii) $\log_e \frac{\partial^2 y}{\partial x \partial y} = x + y.$

(b) Find the general solution of the pair of partial differential equations

$$\frac{\partial u}{\partial x} = x^2 + 3y + e^{x-y}$$
$$\frac{\partial u}{\partial y} = y^2 + 3x - e^{x-y}.$$

(3) (a) Show that the eigen value problem

$$X''(x) + \lambda X(x) = 0, \qquad X'(0) = X'(\pi) = 0, \qquad (0 < x < \pi)$$

has eigen values $\lambda_n = n^2$ and corresponding eigen functions $X_n = \cos nx; n = 0,1,2,...$

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(b) Use the change of variables method to find the general solution, in terms of x, of the differential equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0 \quad ; \qquad (0<x<1).$$

(4) (a) Find the sinusoidal particular solution of the system:

$$\ddot{x}_1+3x_1+x_2=\sin 2t$$
$$\ddot{x}_2+x_1+5x_2=\cos 2t-\sin 2t.$$

(b) Draw the characteristic curves for the partial differential equation

$$-3\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x}+4u=e^{y-3x} \quad ; \text{ where } u=u(x,y).$$

Hence find the general solution of the partial differential equation.

(5) (a) State the conditions satisfied by A,B and C for the second order semi linear partial differential equation ,

$$A(x,y)\frac{\partial^2 u}{\partial x^2}+B(x,y)\frac{\partial^2 u}{\partial x\partial y}+C(x,y)\frac{\partial^2 u}{\partial y^2}=F(x,y)$$

to be classified as hyperbolic , parabolic or elliptic, in the above A, B, C are independent of u and its derivatives.

(b) (i) Using the change of variables

$$\zeta=2y+x^2 \quad , \quad \phi=2y-x^2$$

reduce the equation

$$x^2\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial x^2}-4x^2\frac{\partial u}{\partial y}+\left(4x+\frac{1}{x}\right)\frac{\partial u}{\partial x}=0 \quad ; \quad x\neq 0$$

to the form

$$\frac{\partial^2 u}{\partial q\partial \phi}-\frac{\partial u}{\partial \phi}=0.$$

(ii) Show that the general solution of the above equation is of the form

$$u(x,y)=g\left(x^2+2y\right)+e^{\left(x^2+2y\right)}h\left(2y-x^2\right).$$

(6) (a) Find the general solution of the simultaneous differential equations,

$$\dot{x}_1=8x_1-5x_2+e^t$$
$$\dot{x}_2=-2t+10x_1-7x_2.$$

(b) Consider the above system of equations where $x_1=1$ and $x_2=1$ when $t=0$. Use the Euler method with a step length of 0.1 to calculate approximations to $x_1(0.2)$ and $x_2(0.2)$. Clearly indicate your calculations at each step.