

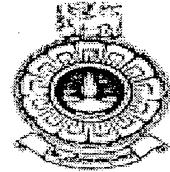
The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme

Final Examination, 2014/2015

Applied Mathematics - Level 05

APU3150/APE5150/AMU3181/AME5181 -Fluid Mechanics



Duration:-Two hours

Date:- 20.10.2015

Time:-9:30a.m.-11:30a.m.

Answer **FOUR** questions (including question 3 and question 6).

1. (a) Briefly describe the velocity vector in each of the following types of flows:
 - i. Steady and Un-steady;
 - ii. Uniform and Non-uniform;
 - iii. Laminar and Turbulent;
 - iv. Compressible and Incompressible;
 - v. Rotational and Irrotational.
- (b) Derive an expression for the acceleration of a fluid particle of velocity \mathbf{q} at point P at time t . Given the velocity $\mathbf{q} = (4 + xy + 2t)\mathbf{i} + 6x^3\mathbf{j} + (3xt^2 + z)\mathbf{k}$, find the acceleration of a fluid particle at $(2, 4, -4)$ at time $t = 3$.
2. (a) Derive the continuity equation for an incompressible fluid flow in the form $\text{div}(\mathbf{q}) = 0$. Deduce the continuity equation, for an incompressible fluid in terms of derivatives of Cartesian components of its velocity vector \mathbf{q} .
- (b) The velocity potential function in a two-dimensional irrotational flow is given by $\phi = (x^2 - y^2)$. Verify that the flow is incompressible and find the velocity components and a stream function for this flow.
- (c) Given $u = x^2y$ and $v = y^2z$ as the two velocity components in the directions of Ox and Oy axes, respectively, find a third component w such that $\mathbf{q} = (u, v, w)$ satisfies the continuity equation.

3. (a) Starting from Euler's equation of motion for a perfect fluid, in the form

$$\underline{\mathbf{F}} - \frac{1}{\rho} \text{grad} p = \frac{\partial \underline{\mathbf{q}}}{\partial t} + \text{grad} \left(\frac{q^2}{2} \right) - \underline{\mathbf{q}} \times \text{curl} \underline{\mathbf{q}}$$

derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

- (b) The cross-section of a horizontal nozzle reduces uniformly from 100 mm bore diameter at the inlet to 50 mm at the exit. It carries a liquid of density 1000 kgm^{-3} at a rate of $0.05 \text{ m}^3\text{s}^{-1}$. The pressure at the wide end is 500 kPa. Calculate the pressure at the narrow end, neglecting friction.

4. (a) Discuss the difference between path line and streamline.

- (b) Suppose in a two-dimensional flow the velocity vector $\underline{\mathbf{q}} = (u_r, v_\theta)^T$ is given by

$$\begin{pmatrix} u_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} krcos2\theta \\ -krsin2\theta \end{pmatrix},$$

for some constant k , in terms of plane polar coordinates. Find a stream function of the flow, and verify that this motion is irrotational.

- (c) Consider the two-dimensional flow with Cartesian components of velocity

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \cos(kx - \alpha t) \end{pmatrix},$$

where u_0, v_0, k and α are constants. Find the particle path and streamline for the particle which passed through the origin, $(x_0, y_0)^T = (0, 0)^T$ at time $t = 0$.

5. (a) For a two-dimensional irrotational motion of an incompressible fluid, write down the components of velocity vector $\underline{q} = u\underline{i} + v\underline{j}$ in terms of stream function and in terms of the velocity potential.
- (b) Suppose a uniform stream of velocity U make an angle θ with positive direction of Ox-axis.
- Write down the complex velocity of the stream. Hence obtain the complex potential of the stream.
 - Deduce from result (i) above, the complex potential of the stream, in the case when the stream flows in the negative Ox-direction.
 - Deduce from result (ii) above, the velocity potential and stream function of the stream.
6. Show that complex potential of a fluid flow due to a two-dimensional sink at the point A where $z = -a$ and a two-dimensional source at the point B where $z = a$, each of strength k , is given by the formula $\Omega(z) = k \ln \left(\frac{z+a}{z-a} \right)$, where a and k are real constants.
- (a) Obtain the equation for the streamlines and equi-potential lines, in this flow. Verify that these two families of curves are mutually orthogonal.
- (b) Show that the fluid speed q at any point P is given by $q = \frac{2ka}{PA \cdot PB}$.

