The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination, 2014/2015
Applied Mathematics - Level 05
APU3150/APE5150/AMU3181/AME5181 -Fluid Mechanics

Duration:-Two hours

Date:- 20.10.2015	**Time:-9:30a.m.-11:30a.m.**

Answer **FOUR** questions (**including question 3 and question 6**).

1. (a) Briefly describe the velocity vector in each of the following types of flows:
   i. Steady and Un-steady;
   ii. Uniform and Non-uniform;
   iii. Laminar and Turbulent;
   iv. Compressible and Incompressible;
   v. Rotational and Irrotational.

   (b) Derive an expression for the acceleration of a fluid particle of velocity $\underline{\mathbf{q}}$ at point P at time t. Given the velocity $\underline{\mathbf{q}} = (4 + xy + 2t)\underline{\mathbf{i}} + 6x^3\underline{\mathbf{j}} + (3xt^2 + z)\underline{\mathbf{k}}$, find the acceleration of a fluid particle at $(2, 4, -4)$ at time $t = 3$.

2. (a) Derive the continuity equation for an incompressible fluid flow in the form $div(\underline{\mathbf{q}}) = 0$. Deduce the continuity equation, for an incompressible fluid in terms of derivatives of Cartesian components of its velocity vector $\underline{\mathbf{q}}$.

   (b) The velocity potential function in a two-dimensional irrotational flow is given by $\phi = (x^2 - y^2)$. Verify that the flow is incompressible and find the velocity components and a stream function for this flow.

   (c) Given $u = x^2y$ and $v = y^2z$ as the two velocity components in the directions of Ox and Oy axes, respectively, find a third component w such that $\underline{\mathbf{q}} = (u, v, w)$ satisfies the continuity equation.

3. (a) Starting from Euler's equation of motion for a perfect fluid, in the form

$$\underline{\mathbf{F}} - \frac{1}{\rho} grad p = \frac{\partial \underline{\mathbf{q}}}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{\mathbf{q}} \times curl \underline{\mathbf{q}}$$

derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

(b) The cross-section of a horizontal nozzle reduces uniformly from 100 mm bore diameter at the inlet to 50 mm at the exit. It carries a liquid of density 1000 kgm^{-3} at a rate of 0.05 m^3s^{-1}. The pressure at the wide end is 500 kPa. Calculate the pressure at the narrow end, neglecting friction.

4. (a) Discuss the difference between path line and streamline.

(b) Suppose in a two-dimensional flow the velocity vector $\underline{\mathbf{q}} = (u_r, v_\theta)^T$ is given by

$$\begin{pmatrix} u_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} krcos2\theta \\ -krsin2\theta \end{pmatrix},$$

for some constant k, in terms of plane polar coordinates. Find a stream function of the flow, and verify that this motion is irrotational.

(c) Consider the two-dimensional flow with Cartesian components of velocity

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 cos(kx - \alpha t) \end{pmatrix},$$

where u_0, v_0, k and α are constants. Find the particle path and streamline for the particle which passed through the origin, $(x_0, y_0)^T = (0,0)^T$ at time $t = 0$.

5. (a) For a two-dimensional irrotational motion of an incompressible fluid, write down the components of velocity vector $\underline{\mathbf{q}} = u\underline{\mathbf{i}} + v\underline{\mathbf{j}}$ in terms of stream function and in terms of the velocity potential.

   (b) Suppose a uniform stream of velocity U make an angle θ with positive direction of Ox-axis.

   i. Write down the complex velocity of the stream. Hence obtain the complex potential of the stream.

   ii. Deduce from result (i) above, the complex potential of the stream, in the case when the stream flows in the negative Ox-direction.

   iii. Deduce from result (ii) above, the velocity potential and stream function of the stream.

6. Show that complex potential of a fluid flow due to a two-dimensional sink at the point A where $z = -a$ and a two-dimensional source at the point B where $z = a$, each of strength k, is given by the formula $\Omega(z) = kln\left(\dfrac{z+a}{z-a}\right)$, where a and k are real constants.

   (a) Obtain the equation for the streamlines and equi-potential lines, in this flow. Verify that these two families of curves are mutually orthogonal.

   (b) Show that the fluid speed q at any point P is given by $q = \dfrac{2ka}{PA \cdot PB}$.

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