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THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme

APPLIED MATHEMATICS-LEVEL 05

AMU3186/AME5186- Quantum Mechanics

FINAL EXAMINATION- 2014/2015

Duration: Two Hours.

Date: 06.11.2015	**Time: 1.30 p.m. to 3.30 p.m.**

Answer **FOUR** Questions only.

(1) (i) Define the commutator [A, B] of two operators $\hat{A}$ and $\hat{B}$.

(ii) If $\hat{A}$, $\hat{B}$,$\hat{C}$ and $\hat{D}$ are operators, show that

(a) $[\hat{A},\hat{B}] = -[\hat{B},\hat{A}]$

(b) $[\hat{A},\hat{B}\,\hat{C}] = [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}]$

(c) $[\hat{A},\hat{B}+\hat{C}+\hat{D}] = [\hat{A},\hat{B}] + [\hat{A},\hat{C}] + [\hat{A},\hat{D}]$

(d) $[\hat{A}+\hat{B},\hat{C}+\hat{D}] = [\hat{A},\hat{C}] + [\hat{A},\hat{D}] + [\hat{B},\hat{C}] + [\hat{B},\hat{D}]$

(iii) Prove that $[\hat{x},\hat{P}_x] = i\hbar$ where a standard notation has been used.

(2) A one dimensional wave function $\psi(x, t)$ is given by

$$\psi(x,t) = B\,Sin\left(\frac{\pi x}{2a}\right)\,e^{-i\alpha t}\quad ;0 \le x \le a \quad \text{where } \alpha \text{ and } a \text{ are constants.}$$

(i) Determine the normalization constant B .

(ii) Calculate the mean vlaues of x, x^2 and $\hat{P}_x$ with respect to $\psi(x)$.

(iii) Calculate (Δx) where Δx has a usual meaning.

(3)An X-ray photon of wave length $\lambda = 10^{-10}\ m$ is incident on stationary electron, where $\lambda_c = \frac{h}{mc}$ is the Compton wave length.

(i) Show that for Compton scattering, $\delta\lambda = \lambda^1 - \lambda = 2\lambda_c\, sin^2\frac{\theta}{2}$, where m is the mass of the electron, λ is the wave length of the incident X-ray and λ^1 is the wave length of X-ray scattered through an angle θ.

(ii) Calculate the Compton shift.

(iii) Calculate the kinetic energy of the recoiling electron if $m = 9.108 \times 10^{-31}\ kg,\ \ c = 3 \times 10^{8}\ ms^{-1}$, $h = 6.625 \times 10^{-34}\ Js$ and $\theta = 30^{o}$.

(4) The parity operator $\hat{\Pi}$ is defined by the operation $\hat{\Pi}\Psi(x) = \Psi(-x)$. Show that

(i) it is a linear operator.

(ii) it is a Hermitian operator.

(iii) Show that $\hat{\Pi}$ commutes with the Hamiltanian $\hat{H} = \frac{\hat{P}^2}{2m} + V(x)$, if the potential energy V is an even function,

(5) The angular momentum of a particle is defined as a vector $\underline{L}$, such that $\underline{L} = \underline{r} \times \underline{p}$, where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle with respect to a fixed origin O.

(i) Write down the Cartesian components $\hat{L}_x, \hat{L}_y, \hat{L}_z$ of the angular momentum operator.

(ii) Hence obtain the angular momentum operator in spherical polar coordinates (r, θ, ϕ). You may use $\hat{\underline{\theta}} = \cos\theta\cos\phi\ \underline{i} + \cos\theta\sin\phi\ \underline{j} - \sin\theta\ \underline{k}$ and $\hat{\underline{\phi}} = -\sin\phi\ \underline{i} + \cos\phi\ \underline{j}$.

(iii) Show that $\left[\hat{L}_y, \hat{L}_z\right] = i\hbar\hat{L}_x$ and $\left[\hat{L}^2, \hat{L}_x\right] = 0$.

(6)(i) If $\hat{A}$ is an operator corresponding to a quantum observable and $\langle\hat{A}\rangle$ is the corresponding expectation value, Show that $\frac{d\langle\hat{A}\rangle}{dt} = \langle\frac{\partial\hat{A}}{\partial t}\rangle + \frac{1}{i\hbar}\langle\left[\hat{A}, \hat{H}\right]\rangle$.

(ii) A particle of mass m and energy E moves in the positive x direction. A square hill Potential is defined by

$$V = \begin{cases} 0; & x < 0 \\ V_0 > 0; & 0 < x < a \\ 0; & a < x \end{cases}$$

Here V_0 is a positive constant

Find the wave function $u(x)$ for each of the regions given above for the case $E < V_0$ and $E > V_0$.

What can you say when $E = V_0$.