The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2014/2015
Applied Mathematics – Level 05

APU 3145/ APE5145 – Newtonian Mechanics II

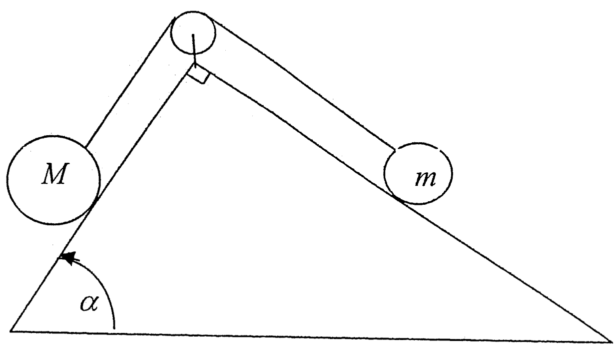
Duration :- Two Hours

Date :-28.10.2015 **Time:-01.30 p.m. 03.30 p.m.**

Answer Four Questions Only.

1. (a) State D' Alembert's principle.

   (b) Two particles of mass M and m are connected by an inelastic string and placed on two fixed inclined planes as shown in the diagram. The coefficient of friction between the particles and the planes is μ. Using D'Alembert's principle, determine the acceleration of the particles and tension in the string.



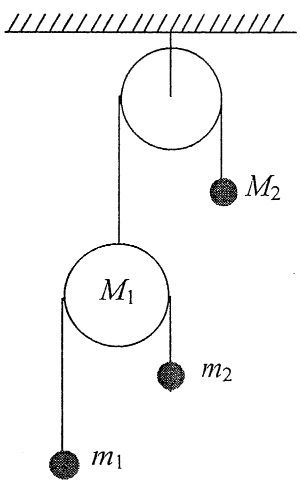
2. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

   (b) A particle of mass m is released from rest from a height h above the surface of the earth. Show that it reaches the earth at a point east of the vertical at a distance $\frac{2}{3}\omega h \cos\lambda \sqrt{\frac{2h}{g}}$ where λ is the attitude of the point of projection and ω is the angular speed of the earth about its polar axis.

03. In the usual notation, derive Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = 1, 2, \ldots, n.$$

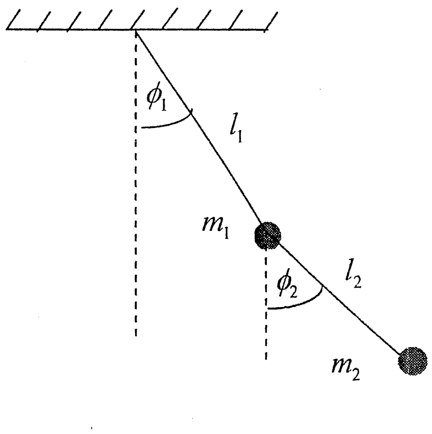
A mass M_2 hangs at one end of a string which passes over a fixed frictionless non-rotating pulley. At the other end of this string there is a non – rotating pulley of mass M_1 over which there is a string carrying masses m_1 and m_2, as shown in the following figure.



(a) Set up the Lagrangian of the system.

(b)Find the acceleration of mass M_2.

4. The double pendulum swinging in a vertical plane consists of two bobs of masses m_1 and m_2 at ends of two weightless rods of lengths l_1 and l_2 and one of them is fixed to a rigid support as shown in figure.



(a) Show that the Lagrangian of the system is given by

$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\phi}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\phi}_2^2 + m_2 l_1 l_2 \dot{\phi}_1\dot{\phi}_2 \cos(\phi_1 - \phi_2) + (m_1 + m_2)\, g l_1 \cos\phi_1 + m_2 g l_2 \cos\phi_2$$

(b) Hence, obtain the Lagrange's equations of motion.

(c) Show further that for the special case $m_1 = m_2 = m$ and $l_1 = l_2 = l$ the Lagrange's equations of motion can be written as

$$2\ddot{\phi}_1 + \ddot{\phi}_2 \cos(\phi_1 - \phi_2) + \dot{\phi}_2^2 \sin(\phi_1 - \phi_2) + 2\frac{g}{l}\sin\phi_1 = 0 \quad \text{and}$$

$$\ddot{\phi}_2 + \ddot{\phi}_1 \cos(\phi_1 - \phi_2) - \dot{\phi}_1^2 \sin(\phi_1 - \phi_2) + \frac{g}{l}\sin\phi_2 = 0.$$

5. (a) Derive Euler's equations of motion for a rigid body rotating about a fixed point.

(b) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body, then show that H and T are conserved.

(c) If a rectangular parallelepiped with its edges $2a$, $2a$ and $2b$ rotates about its center of gravity under no forces, prove that, its angular velocity about one principal axis is constant and about the other axis it is periodic.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) The Hamilton's of a dynamical system is given as

$$H = qp^2 - qp + bp$$

where b is a constant. Write down Hamilton's equations of motion and hence find p, q at time t.