00052

THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
APU3146/APE5146 – OPERATIONS RESEARCH
FINAL EXAMINATION 2014/2015
Duration: Two hours

Date: 02.11.2015	**Time: 09.30 a.m- 11.30 a.m**

Answer four questions only

Question 1

(a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point.

Player B

Player A		B_1	B_2
	A_1	a	$-b$
	A_2	$-c$	d

where a, b, c, d are all non-negative.

Prove that the optimal strategies are:

$$A = \begin{bmatrix} A_1 & A_2 \\ \dfrac{c+d}{a+b+c+d} & \dfrac{a+b}{a+b+c+d} \end{bmatrix} \qquad B = \begin{bmatrix} B_1 & B_2 \\ \dfrac{b+d}{a+b+c+d} & \dfrac{a+c}{a+b+c+d} \end{bmatrix} \text{ and}$$

Value of the game is $v = \dfrac{ad - bc}{a+b+c+d}$.

(b) Sunil is deciding whether or not to make a loan to Nimal who is very poor and who has a bad credit history. Simultaneous to Sunil making this decision, Nimal must decide whether or not to buy gifts for his grandkids. If he buys gifts, he will be unable to repay the loan. If he does not buy gifts, he will repay the loan. If Sunil refuses to give Nimal a loan, then Nimal will have to go to a loan shark. The payoffs in this game are as follows:

If Sunil refuses to make a loan to Nimal and Nimal buys gifts then Nimal gets 7. If Sunil refuses to make a loan to Nimal and Nimal does not buy gifts then Sunil gets 9.If Sunil makes a loan to Nimal and Nimal buys gifts then Sunil gets 5. If Sunil makes a loan to Nimal and he does not buy gifts, then Sunil gets a payoff of 4.

(i) Construct the payoff matrix with respect to Sunil.

(ii) Determine the optimal strategies for Sunil and Nimal.

(iii)Find the value of the game.

Question 2

(a) Briefly explain the following terms:

(i) Arrival pattern

(ii) Service pattern

(iii) Queue discipline

(b) Assume a single channel service system of a library in a school. From past experiences it is known that on an average, every hour 8 students come for issue of the books on an average rate of 10 per hour. Determine the following:

(i) probability of the assistant librarian being idle.

(ii) probability that there are at least 3 students in the system.

(iii)expected time that a student is in queue.

(iv)probability that a student arriving at the library will have to wait.

Question 3

Suppose there are 3 typists in a typing pool. Each typist can type an average of 6 letters per hour. If letters arrive to be typed at the rate of 15 letters per hour,

(i) what fraction of the time are all three typists busy?

(ii) find the average number of letters waiting to be typed.

(iii) what is the probability that one letter in the system?

(iv) determine the average time a letter spends in the system (waiting and being typed).

(v) find the probability a letter will take longer than 20 minutes waiting to be typed and being typed.

Question 4

Suppose people arrive to purchase tickets for a basketball game at the average rate of 4 minutes. It takes an average of 10 seconds to purchase a ticket. If a sports fan arrives 2 minutes before the game starts and if it takes exactly $1\frac{1}{2}$ minutes to reach the correct seat after purchasing a ticket,

(i) can the sports fan expect to be seated for the start of the game? Justify your answer.
(ii) what is the probability that the sports fan will be seated for the start of the game?
(iii) how early must the sports fan arrive in order to be 99% sure of being seated for the start of the game?

Question 5

(i) Define the term inventory.

(ii) What are the advantages and disadvantages of having inventories?

(iii)Formulate the Economic Order Quantity (EOQ) model in which demand is not uniform and production rate is infinite.

Let t_1, t_2,.....,t_n denote the times of successive production runs such that

$t_1 + t_2 + \dots\dots\dots + t_n = 1$ year

(iv) A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the holding cost amount to Rs.0.60 per unit per year. The set up cost per run is Rs. 80. Find the Economic Order Quantity and the total inventory cost.

Question 6

(a) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and replenishment rate is finite.

(b) A factory manufactures a product at the rate of 100 units per day and the daily demand is 40 units. The cost of one unit is Rs. 5. Holding cost is 25% of the value per unit per year. Set up cost is Rs.150 per run.

Using the formula obtained in part(a) answer to the following questions:

(i) determine the economic lot size for one run,

(ii) find the time of cycle,

(iii)find the number of runs per year (Take 250 days for one year) and

(iv)determine minimum total cost for one run.

Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq \text{n}) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad E(v) = \frac{1}{\mu-\lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

(M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1-N\rho^{N-1}+(N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(n) = \frac{\rho\left[1-(N+1)\rho^N+N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ;1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ;n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C \frac{C\mu}{C\mu - \lambda} \right]^{-1} \qquad E(w) = \frac{1}{\lambda} E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; 0 \le n \le C \\ \frac{1}{C^{n-1} C!} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; C < n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C \left\{ 1 - \left(\frac{\lambda}{C\mu} \right)^{N-C+1} \right\} \frac{C\mu}{C\mu - 1} \right]^{-1} & ; \frac{\lambda}{C\mu} \neq 1 \\ \left[\sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu} \right)^C (N - C + 1) \right]^{-1} & ; \frac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2} \left[1 - \rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C} \right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = \left[E(n) \right] / \lambda', \quad \text{where } \lambda' = \lambda(1 - P_N)$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \binom{K}{n} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; \ 0 \le n < R \\ \binom{K}{n} \frac{n!}{R^{n-R} R!} \left(\frac{\lambda}{\mu} \right)^n P_0 & ; \ R \le n \le K \end{cases} \qquad P_0 = \left[\sum_{n=0}^{R-1} \binom{K}{n} \left(\frac{\lambda}{\mu} \right)^n + \sum_{n=R}^{K} \binom{K}{n} \frac{n!}{R^{n-R} R!} \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

$$E(n) = P_0 \left[\sum_{n=0}^{R-1} n \binom{K}{n} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{R!} \sum_{n=R}^{K} n. \binom{K}{n} \frac{n!}{R^{n-R}} \left(\frac{\lambda}{\mu} \right)^n \right] \qquad E(v) = \frac{E(n)}{\lambda \left[K - E(n) \right]}$$

$$E(m) = \sum_{n=R}^{K} (n - R) P_n \qquad E(w) = \frac{E(m)}{\lambda \left[K - E(n) \right]}$$

****