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The Open University of Sri Lanka
Department of Mathematics and Computer Science
B.Sc/ B.Ed Degree Programme
Final Examination - 2014/2015
Applied Mathematics– Level 05
APU3244/ APE5244– Graph Theory

DURATION: - THREE HOURS

Date: - 31 – 10 – 2015 **Time: - 1.30 p.m. – 4.30 p.m.**

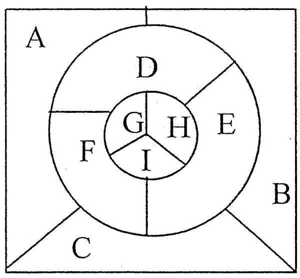
ANSWER FIVE QUESTIONS ONLY

01. (a) Determine the *chromatic index* of each of the following graphs:

(i) C_n for all odd $n \in \mathbb{N}$ (ii) W_n for all $n \geq 4$ (iii) $K_{1,n}$ for all $n \in \mathbb{N}$

(b) Schedule the final examination for the courses *APU1140*, *APU1141*, *APU1142*, *APU2140*, *APU2141*, *APU2142*, *APU2143*, and *APU2144*, using the fewest number of different time slots, if there are no students taking both *APU1140* and *APU2144*, both *APU1141* and *APU2144*, both *APU2140* and *APU2141*, both *APU2140* and *APU2142*, both *APU1140* and *APU1141*, both *APU1140* and *APU1142*, both *APU1142* and *APU2140*, but there are students in every other combination of courses.

(c) Consider the following map.

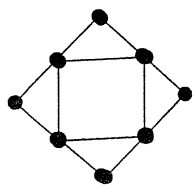


(i) Find the minimum number of colors needs to color the regions of the map so that whenever 2 regions are separated by a boundary takes different color,

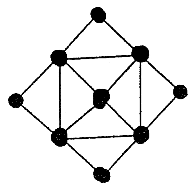
(ii) Draw the planar graph corresponding to this map, and find the minimum number of colors needs to color the vertices of the graph so that adjacent vertices have different colors.

02. (a) If the degree of each vertex of a connected graph G is even, then prove that G is *Eulerian*.

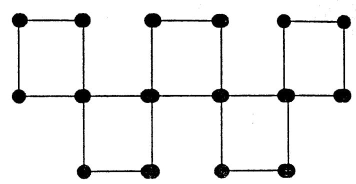
(b) Determine whether each of the following graphs is *Eulerian* or not:



(i)

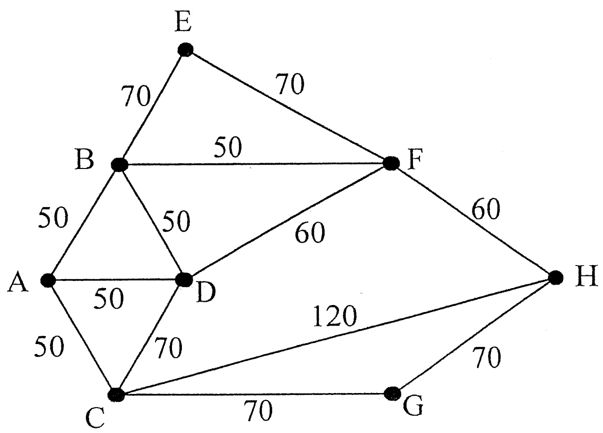


(ii)



(iii)

(c) A municipal council is responsible for maintaining the following roads in a city. The number on each edge is the length of the road in kilometers. The council office is located at A.



(i) A supervisor, based at A, wishes to inspect all the roads. However, the supervisor lives at H and wishes to start his route at A and finish at H. Determine the minimum distance that the supervisor has to travel,

(ii) Use *Dijkstra's algorithm* to find the minimum distance from A to H,

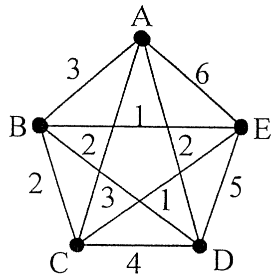
Hence, find the minimum distance that a council worker who also has to monitor all these roads, starting and finishing at A.

03. (a) Disprove each of the following statements by drawing a graph:

(i) All connected graphs have a *Hamiltonian path*,

(ii) All the complete graphs have *Hamiltonian circuit*.

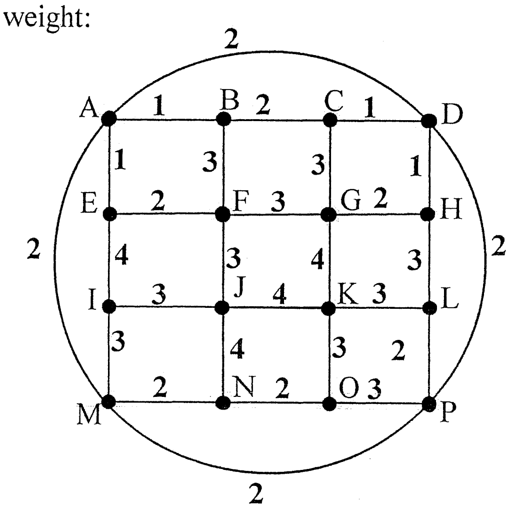
(b) Let G be the following weighted graph.



(i) How many *Hamiltonian cycles* of different weight are in G? Justify for your answer,

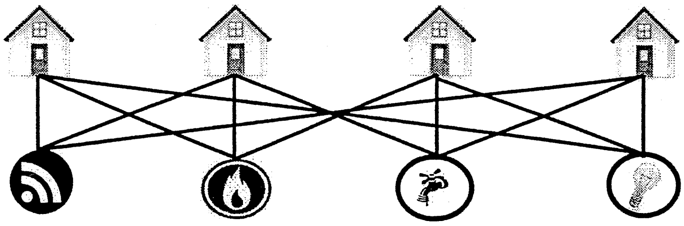
(ii) By deleting the vertex E, find a minimum weighted spanning tree of the remaining graph, Hence, find a solution to the travelling salesman problem.

(c) Use *Kruskal's Greedy algorithm* to find the minimum weighted spanning tree for the following weighted graph and determine its minimum weight:

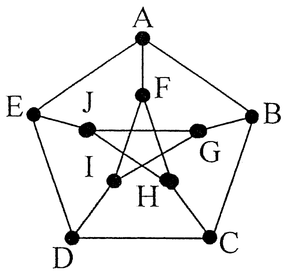


04. (a) Show that a connected simple planar graph whose each vertex has degree at least four must have at least six vertices.

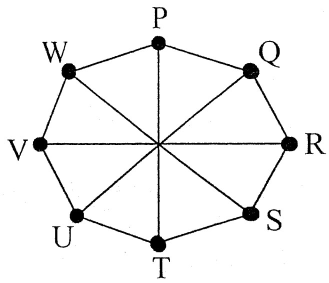
(b) Consider the problem of joining four houses to each of four separate utilities as shown in the following figure. Is it possible to join these houses and utilities so that none of the connection crosses? Justify your answer.



(c) Use the *Kuratowski's theorem* to show that the following graphs are non- plannar.

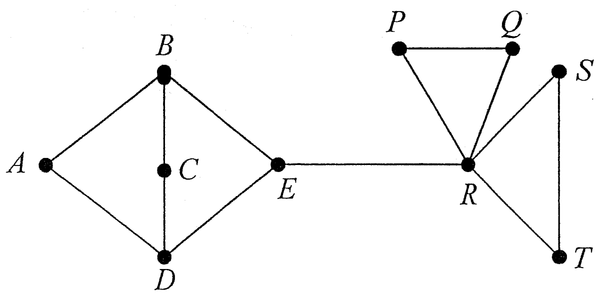


(i)



(ii)

05. (a) Let G be the following graph.



(i) Write down one cut point in G,

(ii) Are there any bridges in G? Justify your answer,

(iii) Find all blocks of G.

(b) (i) Draw the total graph $T(K_3)$ of K_3,

(ii) Draw the line graph $L(K_4)$ of K_4,

Hence, show that $L(K_4)$ is *Eulerian* and *Hamiltonian* but K_4 is non *Eulerian*,

(iii) Determine whether total graph $T(K_3)$ and line graph $L(K_4)$ are isomorphic or not.

Justify your answer.

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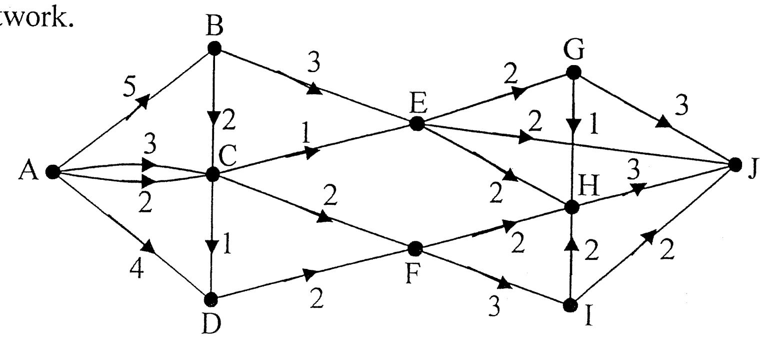
06. Let N be the following network.



(i) Write down all the *edge -disjoint paths* in the above network N ,

(ii) List two AJ - *disconnecting sets* from the above network,

(iii) Draw a possible flow for the network,

(iv) Find the maximum flow of the network,

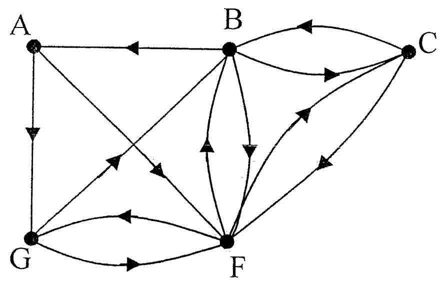
(v) Find the minimum cut of the network,

(vi) Verify the *maximum flow - minimum cut theorem*.

07. Let $X, Y \in V$ and let $d(X, Y)$ be the minimum length among all $X-Y$ walks in a digraph $D=(V, E)$.

(a) Define a strongly connected digraph.

(b) Let $D=(V, E)$ be the following digraph. Write down the adjacency list of D.



(i) Find $d(A, X)$ for all $X \in V$,

(ii) Find $d(X, G)$ for all $X \in V$,

Hence, determine whether D is strongly connected or not,

(iii) Is D a tournament? Justify your answer,

(iv) Verify the *Handshaking dilemma*,

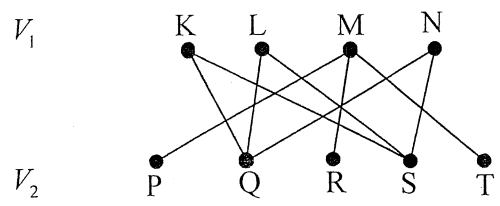
(v) Is $\sum_{i=1}^{n} in\deg(X_i)^2 = \sum_{i=1}^{n} out\deg(X_i)^2$? Justify your answer.

08. Let $G(V_1, V_2)$ be a bipartite graph.

(a) Define a *complete matching* from V_1 to V_2.

(b) State the *Hall's theorem*.

(c) Explain why the following graph has no complete matching from V_1 to V_2.



When does the marriage condition fail in the above graph?

(d) A building contractor advertised for a bricklayer, a carpenter, a plumber and a toolmaker. He received five applicants in which one for the job of bricklayer, one for carpenter, one for bricklayer and plumber, and two for plumber and toolmaker.

(i) Draw the corresponding bipartite graph,

(ii) Show that the marriage condition does not hold for this problem,

(iii) Can all four jobs be filled by the applicants? Justify your answer.