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The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination – 2014/2015
Applied Mathematics – Level 05
AMU3182 /AME5182 – Mathematical Methods-I

Duration: - Two Hours

Date: 16th May 2015 **Time: 1.00 p.m. - 3.00 p.m.**

Answer Four Questions Only.

1. (a) Solve the following homogeneous system:

$$\dot{\underline{x}}(t)=\begin{bmatrix}3&1&-1\\1&3&-1\\3&3&-1\end{bmatrix}\underline{x}(t),\quad \underline{x}(0)=\begin{bmatrix}1\\-2\\-1\end{bmatrix};\text{ where } \underline{x}=[x_1\ x_2\ x_3]^T.$$

(b) Hence or otherwise, write down the general solution of the system

$$\ddot{\underline{x}}(t)=\begin{bmatrix}3&1&-1\\1&3&-1\\3&3&-1\end{bmatrix}\underline{x}(t).$$

2. (a) Let u be a function that satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial y^2}-\frac{1}{y}\frac{\partial u}{\partial y}-4y^2\frac{\partial^2 u}{\partial x^2}=0,\text{ where } y\neq 0.$$

Find u,

(i) if u is a function of x only and
(ii) if u is a function of y only.

(b) Using the change of variables $\varepsilon = x-ct$ and $\phi = x+ct$ reduce the equation $\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=0$, where c is non zero constant, to the form $\frac{\partial^2 u}{\partial \varepsilon \partial \phi}=0$.

3. (a) Show that the boundary value problem

$$u''(x) + 4u'(x) + (4 + \lambda)u(x) = 0, \quad u(0) = u(1) = 0$$

has eigen values $\lambda_n = n^2\pi^2$ with corresponding eigen functions $u_n(x) = e^{-2x} \sin n\pi x, \quad n = 1, 2, \ldots$.

(b) Use a suitable transformation to find the general solution of the equation,

$$x^2 y'' - 6xy' + 10y = 0; \; x > 0 \text{ with } x(1) = -1 \text{ and } y'(1) = 7.$$

4. (a) Find a solution of the system of equations given below:

$$\dot{x}_1 + 10x_1 + 6x_2 - 6e^t = 0$$
$$\dot{x}_2 + 6x_1 + 10x_2 - 4e^t = 0.$$

(b) Consider the following system of equations:

$$\dot{x}_1 = 3tx_2 + 4$$
$$\dot{x}_2 = tx_1 - x_2 - e^t, \quad \text{where } x_1 = 5 \text{ and } x_2 = 2 \text{ at } t = 0.$$

Use the Euler method, with a step length 0.1 to calculate approximations to $x_1(0.2)$ and $x_2(0.2)$.

5. (a) Find a sinusoidal solution of the system of equations given below:

$$\ddot{x}_1 + 4\dot{x}_2 + 3x_1 = 2\sin 3t - \cos 3t$$
$$\ddot{x}_2 + 2\dot{x}_1 - 2x_2 = -3\sin 3t + 2\cos 3t.$$

(b) Find the general solution of the following system of simultaneous partial differential equations,

$$\frac{\partial u}{\partial x} = 3x^2 y - a\sin ax$$
$$\frac{\partial u}{\partial y} = x^3 - e^{-y}.$$

6. (a) Sketch the characteristics curves for the partial differential equation

$$-2\frac{\partial u}{\partial x}-4\frac{\partial u}{\partial y}+5u=e^{x+3y};\ u=u(x,y).$$

Hence, find the general solution of above partial differential equation.

(b) Solve

(i) $y\frac{\partial u}{\partial y}+2xy^2u=y^2$

(ii) $\frac{\partial u}{\partial x}-u\tan x=\cos x.$

where u is, in general, a function of x and y only.