The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2014/2015
Applied Mathematics
AMU3183/AME5183 — Numerical Methods II

Duration: Two Hours

Date: 05. 05. 2015 **Time: 01.00 p.m. – 03.00 p.m.**

Answer Four Questions Only.

1. (a) Prove that

(i) $E = \Delta + 1$, (ii) $E = (1-\nabla)^{-1}$,

where Δ, ∇ and E are the forward difference, the backward difference and the shift operators respectively

(b) Using Newton- Gregory Forward and Backward interpolating polynomials estimate the population in 1895 and 1925 from the following statistics:

Year x	1891	1901	1911	1921	1931
Population y	46	66	81	93	101

2. (a) Evaluate the integral $\int_0^6 \frac{1}{1+x^2}dx$, using

(i) Trapezoidal rule,
(ii) Simpson's One third rule, and also compare the results with its actual value.

(b) Applying Lagrange's interpolation formula, determine $f(5)$ for the following data.

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

3. (a) Derive formula for the Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using Euler's method, solve $\frac{dy}{dx} = 1 + x - y$ with the initial condition $y(1) = 2$ for the range $x = 1.0$ to $x = 2.0$ with $h = 0.2$ correct to 4 decimal places.

4. (a) Derive formula for the modified Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Consider $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 0$. Using Modified Euler's method, compute $y(0.6)$ and $y(0.8)$ correct to 4 decimal places.

5. (a) Using the Taylor series method, solve $\frac{dy}{dx} = x - y^2$, with the initial condition $y(0) = 1$ at $x = 0.1$, correct to 4 decimal places.

(b) Consider $\frac{d^2y}{dx^2} = y - x$, $y(0) = 2$ and $y'(0) = 0$. Using the Taylor series method, find $y(0.2)$ correct to 4 decimal places.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using fourth order Runge-Kutta method, compute $y(0.1)$ and $y(0.2)$ correct to four decimal places given that $\frac{dy}{dx} = -y$, $y(0) = 1$.