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**The Open University of Sri Lanka**
**B.Sc. / B.Ed. Degree Programme – Level 05**
**Final Examination -2014/2015**
**Applied Mathematics**
**APU3240/APE5240 — Numerical Methods**

**Duration: Three Hours**

**Date: 05. 05. 2015** **Time: 1.00 p.m. –4.00 p.m.**

**Answer Five Questions Only.**

1. (a) Derive the Newton- Raphson formula for solving the equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Find the root of the equation $x\ln x-1.2=0$ correct to three decimal places using Newton- Raphson method with $x_0=2$.

2. (a) Derive Newton's divided difference formula.

   (b) Applying Newton's divided difference formula, for the points ( 4, 48), (5, 100), (7, 294), (10, 900), (11. 1210) and (13, 2028) determine $f(x)$ as a polynomial in $x$ and hence find the value of $f(8)$.

3. (a) Show that Cubic Spline interpolation polynomial that passes through the points $(x_0, y_0)$, $(x_1, y_1),\ldots,(x_n, y_n)$ is given by

$$M_{i-1}+4M_i+M_{i+1}=\frac{6}{h^2}\left(y_{i-1}-2y_i+y_{i+1}\right) \text{ where } h=x_{i+1}-x_i,\ f''(x_i)=M_i \text{ and } i=1 \text{ to } n-1.$$

   (b) Find the Cubic Splines interpolation polynomial that passes through the points (1, 4), (2, 1), (4, 3) and (5, 2).

4. (a) Using Newton-Cotes formula or otherwise derive Simpson's One third rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals then show that the error in Simpson's One –Third rule is given by $|E| < \frac{(b-a)h^4}{180} M$, where $M = \max\{y_0^{iv}, y_1^{iv}, y_2^{iv}, \ldots y_{n-1}^{iv}\}$ and $y_i^{iv}$ is the 4th derivative of the $(i+1)^{th}$ ordinate.

(c) Evaluate the integral $\int_0^1 \frac{1}{1+x^2} dx$, using Simpson's One- Third rule, with $h = 0.25$.

5. (a) (i) Derive formula for the Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = x - y^2$, with the initial condition $y(0) = 1$.

(b) Using the Taylor series method, compute $y(0.1)$ and $y(0.2)$ correct to four decimal places given that $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$.

6. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using fourth order Runge-Kutta method, compute $y(0.1)$ and $y(0.2)$ correct to four decimal places given that $\frac{dy}{dx} = -y$, $y(0) = 1$.

7. (a) Show that Milne's Predictor – Corrector formulae to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$ are given by

$$y_{n+1,\,P} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$y_{n+1,\,C} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

(b) It is given that

$$y' = \frac{1}{x+y},\; y(0) = 2,\; y(0.2) = 2.0933,\; y(0.4) = 2.1755 \text{ and } y(0.6) = 2.2493.$$

Find $y(0.8)$, by using Milne's predictor-Corrector method.

8. (a) Show that Adam-Bashforth Predictor – Corrector formulae to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$ are given by

$$y_{n+1,\,P} = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right] \text{ and}$$

$$y_{n+1,\,C} = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right].$$

(b) Given that $y' = y - x^2$, $y(0) = 1$, $y(0.2) = 1.1218$, $y(0.4) = 1.4682$ and $y(0.6) = 1.7379$. Estimate $y(0.8)$ by using Adam-Bashforth Predictor – Corrector method.