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The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2014/2015

APU3143/APE5143-Mathematical Methods

Applied Mathematics -Level 05

Duration: Two Hours.

Date: 25.05.2015	**Time: 1.00 p.m.- 3.00p.m.**

Answer FOUR questions only.

1. The Laplace transform of a function $f(t)$, denoted by $L[f(t)]$, is defined as

$$L[f(t)]=F(s)=\int_0^{\infty} f(t)e^{-st}dt \quad \text{and} \quad L^{-1}\{F(s)=f(t)\}.$$

(a) Find the Laplace transform of the function $f(t)=\dfrac{\sin^2 t}{t}$.

(b) Determine the inverse transform $L^{-1}(F(s))$ of $F(s) = \dfrac{2s+3}{s^3+5s^2+25s+125}$.

(c) Show that $L^{-1}\left\{\dfrac{1}{s^3\left(s^2+1\right)}\right\}=\dfrac{t^2}{2}+\cos t-1$.

(d) Use the convolution theorem to find the inverse Laplace transform of the function

$$\frac{1}{(s+2)^2(s-2)}.$$

2. Solve each of the following boundary value problems using the Laplace transform method:

(a) $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y = 0$, subject to the initial conditions $y(0)= 4$ and $y'(0)=5$.

(b) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = xe^{4x}$, subject to the conditions $y(0) = 5$ and $y'(0) = 0$.

(c) $\dfrac{d^2y}{dx^2} - 4y = e^{-3x} \sin 2x$, subject to the boundary conditions $y(0) = y'(0) = 0$.

3. Find the characteristic values and characteristic functions of the Sturm-Liouville problem given by $\dfrac{d^2y}{dx^2} + \lambda y = 0$; $y(0) = 0$, $y\left(\dfrac{\pi}{2}\right) = 0$.

Show also that the characteristic functions are orthogonal with respect to the weight function having the constant value 1 on the interval $0 \le x \le \dfrac{\pi}{2}$.

4. (a) Consider the function $f(x)$ defined by

$$f(x) = x^2, \quad -\pi \le x \le \pi$$

Find the trigonometric Fourier series of $f(x)$ in $-\pi \le x \le \pi$.

(b) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} 0 & ; 0 \le x \le \pi/2 \\ 2 & ; \dfrac{\pi}{2} < x \le \pi \end{cases}$$

Find the Fourier sine series and the Fourier cosine series of $f(x)$ on $0 \le x \le \pi$.

5. (a) The Gammá function, denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p) = \int_0^\infty e^{-t} t^{p-1} dt$, $(p > 0)$.

Evaluate each of the following integrals using Gamma function:

(i) $\int_0^\infty a^{-bx^2} dx$; where a and b are positive constants.

(ii) $\int_0^\infty 3^{-4x^2} dx$.

(b) The Beta function denoted by $\beta(p,q)$ is defined by $\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}\,dx,$

where $p>0$ and $q>0$ are parameters.

Prove each of the following results using Beta function:

(i) $\left[\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\,d\theta\right]\left[\int_0^{\frac{\pi}{2}}(\sin\theta)^{-\frac{1}{2}}\,d\theta\right]=\pi.$

(ii) $\int_0^{\frac{\pi}{2}}\sin^7\theta.\cos^7\theta\,d\theta=\frac{1}{280}.$

6. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x)=x^p\sum_{m=0}^{\infty}\frac{(-1)^m x^{2m}}{2^{2m+p}.m!\Gamma(p+m+1)}.$$

(a) Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(b) Show that $\frac{d}{dx}\{xJ_n.J_{n+1}\}=x\left[J_n^2-J_{n+1}^2\right].$

(c) Prove that $J_0''=\frac{1}{2}\left[J_2-J_0\right]$ where "denotes the standard notation.