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**The Open University of Sri Lanka**

**B.Sc./B.Ed. Degree Programme-Level-05**

**Department of Mathematics and Computer Science**

**Final Examination- 2014/2015**

**Computer Science**

**CPU3140- Mathematics for Computing**

**Duration: Two Hours**

| **Date :30.04.2015** | **Time: 9.30am-11.30am** |
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**Answer Four Questions only.**

**01.**(i) Reduce the following sentences to statement form.

(a) Grass will grow only if enough moisture is available.

(b) A necessary condition for $x$ to be prime is that $x$ is odd or $x=2$.

(c) A sufficient condition for $f$ to be continuous is that $f$ is differentiable.

(d) It is raining but the sun is still shining.

(e) If taxes are increased or government spending decreases, then inflation will not occur this year.

(ii) Using truth tables prove the following statements.

(a) $(A \cup \sim B) \rightarrow (C \cap A)$

(b) $(A \cap \sim B) \leftrightarrow (B \rightarrow A)$

(iii) Find which of the followings are tautologies or contradictions.

(Show all your workings).

(a) $(A \leftrightarrow (A \cap \sim A)) \leftrightarrow \sim A$

(b) $((A \to B) \to C) \to ((C \to A) \to (B \to A))$

(c) $(A \cup B) \cap (A \cup \sim B) \cap (\sim A \cup B) \cap (\sim A \cup \sim B)$

**02.** (i) Define the following matrices.

(a) A square matrix

(b) A rectangular matrix

(c) A diagonal matrix

(d) An identity matrix

(ii) If $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ -1 & 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$

Find the values of $x$, $y$ and $z$

(iii) If $C = \begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$ and $D = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$

Find CD.

(iv) If A and B are two matrices and if they commute, then show That $(AB)^n = A^n B^n$, for all $n \in \mathbb{N}$ (the set of natural numbers).

**03.** (i) What is meant by a linear recurrence relation?

(ii) Name the types of linear recurrence relations.

(iii) Classify the following recurrence relations according to the types that you have given in part(ii)

(a) $P_n = (1.11)P_{n-1}$
(b) $a_n = a_{n-1} + a_{n-2}^2$
(c) $H_n = 2H_{n-1} + 1$

(iv) Find the solution of the recurrence relation given below.
$a_n = 2a_{n-1} - 2a_{n-2}$ with $a_0 = 1$ , $a_1 = 2$

(v) Using your answer to part (iv) ,find the solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2} + 2^n$ for n≥2 with $a_0 = 1$ ,$a_1 = 2$

(vi) Using your method to part (iv) find the particular solution of the recurrence relation $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0 = 8$, $a_1 = 6$ and $a_2 = 26$

**04.** (i) Let A= {1,2,3,4,5} ,B= {1,2,3} and C={ 4,5} . List the elements of the following sets.

(a) (A ∪ B )
(b) (A− B)
(c) $\bar{\bar{A}}$
(d) ( A Δ C)

(ii) A,B,C are subsets of U and U is the universal set. Verify the following statements using Venn diagrams.

(a) $\overline{A \cup B} = \bar{A} \cap \bar{B}$
(b) A− (B ∪ C) = (A−~B)∩(A−~C)
(c) $(A \cup B)^c \cup (A \cap B) = ((A - B) \cup (B - A))^c$

(iii) The relation $R \subseteq \mathbb{N} x\mathbb{N}$ is determined by $aRb \leftrightarrow b=a^x$ for some nonzero $x \in \mathbb{Q} \setminus \{0\}$. Prove that R is an equivalence relation.

**05.** (a) Given two real numbers $a$ and $b$. Give the names of the intervals and write these in set notation.

(i) [a, b]
(ii) (a, b)
(iii) (a, b ]
(iv) [a, ∞ )

( b ) (i) What do you mean by a function? what are the main components that you need to define a function?

(ii) Give a real life situation to explain a function.

(iii) $f(x) = x^2$ *and* $g=x+1$ , find $f \, 0g$ *and* $g \, 0f$. Are these two the same? What does this information tell you about composition?

( c ) (i) Let a function $f$ is given by $f(x)$ =-x+2. What is the value of $f(10)$ ?

(ii) If $f(x)= \sqrt{x}$ and $g(x) = x^2+1$ , find a formula for the composition $f \, 0 \, g$.

(iii) Find the inverse of the composite function that you composed in part (ii).

**06.** Let G be a graph with set of four vertices $\{v_1, v_2, v_3, v_4\}$, whose adjacency matrix A is given by

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(i) Without drawing the diagram of G, determine whether G is connected.

(ii) Find the number of paths of length three joining $v_2$ & $v_4$ and name all those paths.

(iii) Write down all the components of G.

(iv) Is G a forest ? Justify your answer.