The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme-Level-05

Department of Mathematics and Computer Science

Final Examination- 2014/2015

Pure Mathematics/Computer Science

PMU3294/CSU3276/PME5294-Discrete Mathematics

Duration: Three Hours

uration: Inree Ho

Time: 9.30am-12.30pm

Date: 30.04.2015

Answer Five Questions Only

O1.(a) Let p and q be two statements . Use truth tables to determine whether each of the following statements is a tautology, a contradiction or a contingency:

(i)
$$[\sim q \cap (p \rightarrow q)] \rightarrow p$$

(ii)
$$(p \cap \sim q) \cup (q \cap \sim p)$$

(iii)
$$p \cap (p \rightarrow q) \cap \sim q$$

- (b) Write the inverse and converse of the following statements:
 - (i) "If the density of a fluid is not $1000 \text{kg/}m^3$, then the fluid cannot be water".
 - (ii) " If $\sqrt{2}$ is rational, then either $\sqrt{2}$ is algebraic or $\sqrt{2}$ is transcendential".
- (c) Let *p* be "It is cold" and let *q* be "It is raining". Give a simple verbal sentence which describes each of the following statements:

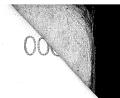
(ii)
$$p \cap q$$

(iii)
$$p \cup q$$

(iv)
$$q \cup \sim p$$

02. Let G be a graph with set of four vertices $\{v_1, v_2, v_3, v_4\}$, whose adjacency matrix A is given by

- (i) Without drawing the diagram of G, determine whether G is connected.
- (ii) Find the number of paths of length three joining $v_2 \ \& \ v_4$, and name all those paths.
- (iii) Write down all the components of G.
- (iv) Is G a forest? Justify your answer.



- 03. A person invests Rs 20000/= at 15 % interest compounded annually. If A_n represents the amount at the end of n years, find
 - (i) a difference equation satisfied by A_n and the initial conditions that define the sequence $\{A_n\}$
 - (ii) an explicit formula for A_n . Hence, find the time it takes for the person to double the initial Investment.
 - (iii) Find the general solution of the difference equation.

$$f(n+2) - 4 f(n) = n(1+3^n)$$

- 04.(a) Prove that the number of ways in which n distinct objects can be distributed into k boxes, $B_1, B_2, ..., B_k$, such that there are r, objects in box B, for i=1,2,3,.....k, is $\frac{n!}{(n_1! \, n_2! \, n_3! \, \, n_k!)}$
 - (b) (i) Find the number of ways that seven toys can be divided among three children if the youngest child is to receive three toys and each of the others two toys.
 - (ii) A box B contains seven marbles numbered 1 through 7. Find the number of ways of drawing from B first two marbles, then three marbles, and lastly the remaining two marbles.
 - (c) A group of 5 students is selected from 12 eligible students in a campus to attend a conference. In how many ways can the group be chosen
 - (i) if 2 of the eligible students will not attend the conference together?
 - (ii) if 2 of the eligible students are married and will only attend the conference together?
- 05. (a) Let A and B be two events with P(A) > 0. Define P(B/A), the conditional probability of B given A.
 - (b) Find P(B/A) if,
 - (i) A is a subset of B,
 - (ii) A and B are mutually exclusive.
 - (c) In a certain college 25% of the students failed mathematics ,15% of the students failed computer science, and 10% of the students failed both mathematics and computer science. A student is selected at random.
 - (i) If he failed computer science, what is the probability that he failed mathematics?
 - (ii) What is the probability that he failed mathematics or computer science,?
 - (iii) Determine whether the event failed mathematics is depend on the event failed computer science.

- 06.(a) Define each of the following terms:
 - (i)Binary Relation, (ii) Partial order, (iii) Total order, (iv) Equivalence Relation
 - (b) Let $X=\{1,2,3,4\}$ and let $R=\{(1,1), (2,2), (3,3), (4,4), (1,2), (2,3), (1,3)\}$. Prove that R is a partial order on X.
 - (c) Let S and T be partial orders on a nonempty set Y. Does it follows that $S \cup T$ is a partial order on Y? Justify your answer.
 - (d) Define the relation S on the set \mathbb{R} of all real numbers by for each $x, y \in \mathbb{R}$, xSy if x-y for some $m \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$. Prove that S is an equivalence relation on \mathbb{R} .
- 07. (a) What are the postulates that should be true for a nonempty set G of elements to be a group under the binary operation *.
 - (b) Define an abelian group, a homomorphism and an isomorphism.
 - (c)(i)Use a Cayley composition table to show that the set of functions $G = \{x, -x, \frac{1}{x}, -\frac{1}{x}\}$ under the binary operation of composition of functions forms an abelian group.
 - (ii)Let G be the group of real numbers under the usual addition, and let G' be the group of positive real numbers under the usual multiplication.

Show that the mapping $f: G \rightarrow G'$, defined by

 $f(a) = 2^a$, is a homomorphism.

Is it an isomorphism? Justify your answer

- 08. Prove or disprove each of the following statements, and name the method of your proof in each case:
 - (a) Every continues function is differentiable,
 - (b) For each $n \in \mathbb{N}$, $17^n 10^n$ is divisible by 7,
 - (c) $\sum_{n=1}^{\infty} r^n$ is divergent implies that $|r| \ge 1$,
 - (d) There exists $x \in \mathbb{R}$ such that $x^{i\pi} + 1 = 0$,
 - (e) Let a, b be real numbers. If $a + b \ge 6$ then $a \ge 3$ or $b \ge 3$.

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