The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2015/2016
Applied Mathematics
APU3240/APE5240 — Numerical Methods

Duration: Three Hours

Date: 03. 07. 2016 **Time: 01.00 p.m. – 04.00 p.m.**

Answer Five Questions Only.

1. (a) Derive Newton- Raphson formula for solving the equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Using Newton- Raphson method, taking initial given x_0 as 0.4, find the root of the equation $3x-\sqrt{1+\sin x}=0$ correct to four decimal places.

2. (a) Prove that

   (i) $E=\Delta+1$, (ii) $E=(1-\nabla)^{-1}$, (iii) $\delta=E^{1/2}-E^{-1/2}$,

   (iv) $\nabla\Delta=\Delta\nabla=\delta^2$ (v) $\Delta-\nabla=\delta^2$

   where Δ, ∇, δ and E are the forward difference, the backward difference, the central difference and the shift operators respectively.

   (b) Derive Gregory- Newton forward interpolation formula.

   (c) Hence, interpolate $f(45)$, given that $f(x)$ passes through the points (40, 31), (50, 73), (60, 124), (70, 159) and (80, 190).

3. (a) (i) Derive Newton's general interpolation formula with divided differences.

(ii) Hence, find the value of $f(8)$, given that $f(x)$ passes through the points

(4, 48), (5, 100), (7, 294), (10, 900), (11, 1210) and (13, 2028).

(b) (i) Derive Lagrange's interpolation formula.

(ii) Applying Lagrange's formula inversely, obtain the root of the equation $f(x)=0$, given that $f(30)=-30,\ f(34)=-13,\ f(38)=3$ and $f(42)=18$.

4. (a) Derive Simpson's three eight's Rule.

(b) If the interval $[a, b]$ is divided into $3n$ sub intervals then show that the error in Simpson's three eight's rule is given by $|E|<\frac{(b-a)h^4}{80}M$ where M is the numerically greater value of $y_0^{iv},\ y_3^{iv},\ \cdots y_{3n-3}^{iv}$.

(c) Applying Simpson's three eight's rule for following data,

sin0	$\sin(\pi/12)$	$\sin(\pi/6)$	$\sin(\pi/4)$	$\sin(\pi/3)$	$\sin(5\pi/12)$	$\sin(\pi/2)$
0	0.2588	0.5000	0.7071	0.8660	0.9659	1.0

evaluate the integral $\int_0^{\pi/2} \sin x\, dx$.

5. (a) (i) Derive formula for Picard's method to solve $\frac{dy}{dx}=f(x,y)$ subject to the initial condition $y(x_0)=y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx}=3e^x+2y$ with the initial condition $y(0)=0$.

(b) (i) Derive formula for Euler's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Solve $\frac{dy}{dx} = 3x^2 + 1$ with the initial condition $y(1) = 2$ using the Euler's method. Estimate $y(2)$ taking $h = 0.25$.

6. (a) Applying Taylor series method of fourth order for the differential equation

$\frac{dy}{dx} = xy^2 + 1$ subject to the initial condition $y(0) = 1$, evaluate $y(0.2)$ and $y(0.4)$.

(b) Applying Taylor series method of fourth order for the system of differential equations $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$ subject to the initial conditions $y(0) = 2$ and $z(0) = 1$, evaluate $y(0.1)$, $y(0.2)$, $z(0.1)$ and $z(0.2)$.

7. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = 1 + y^2$ with the initial condition $y(0) = 0$ using Runge-Kutta method of fourth order. Evaluate the value of y, when $x = 0.2$ and $x = 0.4$.

(c) Solve $\frac{d^2y}{dx^2} = y^3$ with the initial condition $y(0) = 10$, $y'(0) = 5$ using Runge-Kutta method of fourth order and evaluate $y(0.1)$.

8. (a) State Milne's Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using Taylor series method, solve $\frac{dy}{dx} = xy + y^2$ with the initial condition $y(0) = 1$.

Hence find $y(0.1)$, $y(0.2)$ and $y(0.3)$. Now find $y(0.4)$ by using Milne's Predictor – Corrector Method.