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The Open University of Sri Lanka

B.Sc./B.Ed Degree Programme-Level-05

Department of Mathematics and Computer Science

Final Examination- 2015/2016

Pure Mathematics

PMU3294/CSU3276/PME5294-Discrete Mathematics

Duration: Three Hours

Date: 02.07.2016	Time: 9.30am-12.30pm

Answer Five Questions Only

01.(a) Let p and q be two statements . Use the truth tables to determine whether each of the following statement is tautology, contradiction or contingency.

(i) $[\sim q \cap (p \rightarrow q)] \rightarrow p$

(ii) $(p \cap \sim q) \cup (q \cap \sim p)$

(iii) $p \cap (p \rightarrow q) \cap \sim q$

(b) Write the inverse and converse of the following statements.

(i) "If the density of a fluid is not 1000kg/m^3 then the fluid cannot be water".

(ii)" If $\sqrt{2}$ is rational then either $\sqrt{2}$ is algebraic or $\sqrt{2}$ is transcendential ".

(c) Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements:

(i) $\sim p$

(ii) $p \cap q$

(iii) $p \cup q$

(iv) $q \cup \sim p$

02. Let G be a graph with set of four vertices $\{v_1, v_2, v_3, v_4\}$, whose adjacency matrix A is given by

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(i) Without drawing the diagram of G, determine whether G is connected.

(ii)Find the number of paths of length three joining v_2 & v_4 and name all those paths.

(iii)Write down all the components of G.

(iv) Is G a forest ? Justify your answer.

03. A person invests Rs 2000/= at 15 percent interest compounded annually. If $A_n$represents the amount at the end of n years find

(i) a difference equation satisfied by A_n and the initial conditions that define the sequence $\{A_n\}$

(ii) an explicit formula for A_n. Hence, deduce that, how long will it takes for the person to double the initial investment?

(iii) Find the general solution of the difference equation given below.

$f(n+2) -4 f(n)= n(1+3^n)$

04.(a) Prove that the number of ways in which n distinct objects can be distributed into k boxes, $B_1, B_2, \ldots\ldots. B_k,$ such that there are r_i, objects in box B_i, for i=1,2,3,.........k,is $n!/(n_1!\, n_2!\, n_3! \ldots\ldots n_k!)!$

(b) (i) Find the number of ways that seven toys can be divided among three children if the youngest child is to receive three toys and each of the others two toys.

(ii) Let a box contain seven marbles numbered 1 through 7. Find the number of ways of drawing from B first two marbles, then three marbles, and lastly the remaining two marbles.

(c) A group of 5 students is selected from 12 eligible students in a campus to attend a conference.

(i) In how many ways can the group be chosen?

(ii) In how many ways if 2 of the eligible students will not attend the conference together?

(iii) In how many ways if 2 of the eligible students are married and will only attend the conference together?

05. (a) Let A and B be two events with P (A) >0. Define P(*B/A*), the conditional probability of B given A.

(b) Find P(*B/A*) if,

(i) *A* is a subset of *B*,

(ii) *A* and *B* are mutually exclusive.

(c) In a certain college 25% of the students failed mathematics ,15% of the students failed computer science, and 10% of the students failed mathematics and computer science. A student is selected at random.

(i) If he failed computer science, what is the probability that he failed mathematics,

(ii) What is the probability that he failed mathematics or computer science,

(iii) Determine whether the event failed mathematics is depend on the event failed computer science.

06. (i) Define the following terms:

(a)Binary Relation, (b) Partial order , (c) Total order, (d) Equivalence Relation

(ii) Let X={1,2,3,4} and let R ={(1,1), (2,2), (3,3), (4,4), (1,2), (2,3), (1,3)}. Prove that R is a partial order on X.

(iii) Let S and T be partial order on a nonempty set Y. Does it follows that S ∪ T is a partial order on Y. Justify your answer.

(iv)Define the relation S on the set $\mathbb{R}$ of all real numbers by for each x, y $\in \mathbb{R}$, *xSy* if x-y $=\frac{m}{2^n}$ for some $m \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$. Prove that S is an equivalence relation on $\mathbb{R}$.

07. (a)What are the postulates that should be true for a nonempty set G of elements to be a group under the binary operation *.

(b)Define an abelian group, homomorphism and isomorphism.

(c)(i)Use a Cayley composition table to show that the set of functions G={x,-x, $\frac{1}{x}$, $-\frac{1}{x}$} under the binary operation of composition of functions forms an abelian group.

(ii)Let G be the group of real numbers under the usual addition, and let *G'* be the group of positive real numbers under the usual multiplication. Show that the mapping f: G → G',defined by

$f(a)=2^a$, is a homomorphism.

Is it an isomorphism? Justify your answer

08. Prove or disprove each of the following statements and name the method of your proof in each case.

(a) Every continuous function is differentiable,

(b) For each $n \in N$, $17^n - 10^n$ is divisible by 7,

(c) $\sum_{n=1}^{\infty} r^n$ is divergent implies that $|r| \geq 1$,

(d) For each $x \in \mathbb{R}$, for each $y \in \mathbb{R}$, $\frac{x+y}{2} \geq \sqrt{xy}$,

(e) There exists $x \in \mathbb{R}$ such that $x^{i\pi} + 1 = 0$,

(f) Let a , b be real numbers. If $a+b \geq 6$ then $a \geq 3$ or $b \geq 3$.