

THE OPEN UNIVERSITY OF SRILANKA
 BACHELOR OF SCIENCE DEGREE PROGRAMME – Level 05
 DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE
 FINAL EXAMINATION 2015/2016
 CPU 3140 – MATHEMATICS FOR COMPUTING
 DURATION : TWO HOURS (2 HOURS)



DATE : 02.07.2016

TIME : 9.30 am – 11.30 am

Answer **FOUR** Questions Only.

1.

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

I. Find A^2

II. Suppose $f(x) = 2x^2 - 3x + 5$. Find $f(A)$.

b) Given that $B = \begin{bmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{bmatrix}$

I. Find the matrix BB^T .

II. Hence, find the inverse of the matrix **B**.

c) Compute the determinant of each of the following matrices.

I. $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}$

II. $B = \begin{bmatrix} 4 & -6 & 8 & 9 \\ 0 & -2 & 7 & -3 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

III. $C = \begin{bmatrix} 1/2 & -1 & -1/3 \\ 3/4 & 1/2 & -1 \\ 1 & -4 & 1 \end{bmatrix}$

2.

- I. Prove the following identities, stating carefully the set laws you are using at each stage of the proof.
 - a) $B \cup (\emptyset \cap A) = B$
 - b) $(A \cap B) \cup (\cap B') = A$
 - c) $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$
- II. Give the interval notation to represent the sets given below.
 - a) The set of all numbers from 2 through 7 inclusive.
 - b) The set of all numbers greater than or equal to 5 and less than 12.
 - c) The set of all real numbers greater than -6 and less than or equal to 14.
- III. Give the inequality notation or set builder notation to describe the following sets.
 - a) $\{-3, -2, -1, 0, 1, 2\}$
 - b) $\{1, 2, 3, 4\}$
 - c) $\{0, 1, 2, 3\}$
- IV. In a group of 60 people, 27 like to drink tea and 42 like to drink coffee, and each person likes at least one of the two drinks. How many like both tea and coffee?

3.

- I. Functions f and g are given by $f(x) = (x-2)$ and $g(x) = 3x^2 + 1$; $x \in \mathbb{R}$.
 - a) Find $f \circ g(x)$ and $g \circ f(x)$.
 - b) Are these two answers the same? What does this information tell you about compositions of function?
- II. Given that $f(x) = x^2$, $g(x) = 2x$ and $h(x) = (x-2)$; $x \in \mathbb{R}$.
 - a) Find $(f \circ g) \circ h(x)$ and $(f \circ (g \circ h))(x)$.
 - b) Are these two answers the same? What does this information tell you about compositions of functions?
- III. The formula $K(C) = C + 273$ converts Celsius temperature to Kelvin .
The formula $C(F) = \frac{5}{9}(F - 32)$ converts Fahrenheit temperature to Celsius.
 - a) Write a composition function that would convert Fahrenheit temperature to Kelvin.
 - b) Using the above formula convert the boiling point of water (212°F) and the freezing point of water (32°F) to Kelvin.

4.

- I. What are the three basic stages that you consider when using Mathematical Induction?
- II. Use Mathematical Induction to prove that $1^3+2^3+3^3+\dots+n^3 = n^2/4(n+1)^2 \forall n \in \mathbb{N}$.
- III. Prove that for any positive integer number n , n^3+2n is divisible by 3.

5.

- I. Classify the following recurrence relations, if possible.

a) $a_{n+1} = 5a_n$

b) $a_{n+3} + 5a_{n+2} + 4a_{n+1} + a_n = \cos n$

c) $a_n a_{n-1} + a_{n-1} a_{n-2} = a^2$

- II Find the solutions of the following recurrence relations.

a) $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$.

b) $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$

- III A person deposits Rs 10,000/= in a savings account at a bank yielding 5% per year with interest compounded annually. How much money will be in the account after 30 years?

6.

- I Let p and q be two statements. Use the truth tables to determine whether each of the following statement is tautology, contradiction or contingency.

a) $[\sim q \wedge (p \rightarrow q)] \rightarrow p$

b) $(p \wedge \sim q) \cup (q \wedge \sim p)$

c) $p \wedge (p \rightarrow q) \wedge \sim q$

- II Write the inverse and converse of the following statements.

a) "If the density of a fluid is not 1000kg/m^3 then the fluid cannot be water".

b) "If $\sqrt{2}$ is rational then either $\sqrt{2}$ is algebraic or $\sqrt{2}$ is transcendental".

- III Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence in English which describes each of the following statements;

a) $\sim p$

b) $p \wedge q$

c) $p \vee q$

d) $q \vee \sim p$