The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Open Book Test- 2015/2016
Applied Mathematics- Level 04
APU2143/APE4143 — Vector Calculus

Duration: One Hour

Date: -27.03. 2016 **Time:– 10:30a.m. - 11:30a.m.**

ANSWER ALL QUESTIONS.

1. (a) State and sketch the domain of the function $f(x, y) = \sqrt{x} + \sqrt{y}$.

   (b) Sketch the level curves of the function $f(x, y) = 1 - 4x^2 - 4y^2$.

   (c) Find the value of the following limits, if they exist:

   (i) $\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$, (ii) $\lim_{(x,y)\to(0,0)} \frac{y^4}{x^2 + y^2}$.

   (d) Discuss the continuity of the following function at the origin:

$$f(x, y) = \begin{cases} \frac{y^4}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

2. (a) If $z = f(x, y)$ where $x = uv$ and $y = u^2 - v^2$ then show that

   (i) $2x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v}$,

   (ii) $2\frac{\partial z}{\partial v} = \frac{1}{u^2 + v^2}\left(u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v}\right)$.

   (b) Find the equation of the tangent plane to the surface $z = x^2 + y^3$ at the point (1, 1, 2).

   (c) In which direction does the function $f(x, y) = xy^2 + x^3 y$ increase most rapidly at the point (1, 2)?
   In which direction does it decrease most rapidly at point (1, 2)?

   (d) Find all local maxima and minima of the function $f(x, y) = y^2 + xy + 2x + 3y + 6$ and determine their nature.