The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Applied Mathematics – Level 05

APU3143/APE5143 – Mathematical Methods

No Book Test (NBT) – 2015/2016

DURATION: ONE HOUR

Date: 30.04.2016. **Time: 16:00h – 17:00h**

ANSWER **ALL** QUESTIONS.

1. The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p)=\int_0^\infty e^{-t}t^{p-1}dt, \quad (p>0).$

 (a) Using the result $\Gamma(p+1)=p\Gamma(p)$, Compute each of the following:

 (i) $\dfrac{\Gamma 4.\Gamma 3}{\Gamma 4.5}$ (ii) $\Gamma(-4.3)$

 (b) When n is a positive integer and $m>1$, prove that $\int_0^1 x^{m}(ln\ x)^n dx=\dfrac{(-1)^n}{(m+1)^{n+1}}\Gamma(n+1)$

 (c) The Beta function denoted by $\beta(p,q)$ is defined by $\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dx,$

 where $p>0$ and $q>0$ are parameters.

 Prove that $\int_{-1}^1 \left(1-t^2\right)^n dt=\dfrac{2^{n+1}.n!}{1.3.5...(2n+1)}$ for $n=0, 1, 2,...$

2. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p}.m!\Gamma(p+m+1)}.$$

(a) Prove each of the following:

(i) $\frac{d}{dx}\{J_p(x)\} = J_{p-1}(x) - \frac{p}{x}J_p(x)$ or $xJ'_p(x) = xJ_{p-1}(x) - pJ_p(x)$

(ii) $J'_p(x) = \frac{p}{x}J_p(x) - J_{p+1}(x)$

(Hint: You may use the following recurrence relations, if necessary without proof.)

$$\frac{d}{dx}\{x^p J_p(x)\} = x^p J_{p-1}(x)$$

$$\frac{d}{dx}\{x^{-p} J_p(x)\} = -x^{-p} J_{p+1}(x)$$

(b) Prove that $\frac{d}{dx}\{J_n^2(x)\} = \frac{x}{2n}\left[J_{n-1}^2(x) - J_{n+1}^2(x)\right]$

(c) Evaluate $\int x^3 J_0(x)\,dx$.