

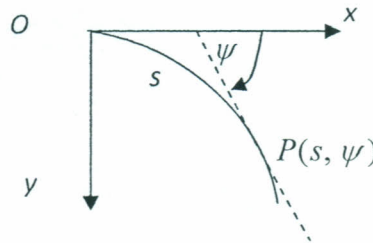


Date:- 08. 10. 2016

Time:- 9:00 To 10:00

Answer All Questions.

1. The figure shows a curve  $C$  that forms the vertical cross-section of a smooth surface. A particle  $P$  moves in a vertical plane along the curve  $C$ , whose intrinsic equation is:  
 $s = a \tan \psi$ ,  $0 \leq \psi < \pi/2$ .



The coordinates  $(s, \psi)$  of  $P$  are measured relative to a fixed point  $O$  and a fixed horizontal line  $Ox$ . The particle is released from rest from the point where  $\psi = \pi/6$  and slides down the surface along  $C$ .

- (a) Show that, while the particle remains in contact with the surface, the speed  $v$  of the particle is given by  $v^2 = 2ga \left( \sec \psi - \frac{2}{\sqrt{3}} \right)$ .
- (b) Show that the particle leaves the surface after it has travelled a distance  $(\sqrt{39} - \sqrt{3})a/3$  along the curve.
2. A particle  $A$ , of mass  $m$ , is held at rest on a smooth horizontal table. One end of a light inextensible string of length  $l$  is attached to  $A$ . The string passes through a small smooth hole  $H$  in the table, and carries a particle  $B$ , of mass  $m$ , hanging freely at the other end. Initially  $HA = a$  and the particle  $A$  is moving horizontally with speed  $2\sqrt{ag}$  in a direction perpendicular to  $HA$ .
- (a) If  $r$  is the distance  $HA$  after time  $t$ , show that  $\dot{r}^2 = \frac{g}{r^2} (r - a)(2a^2 + 2ar - r^2)$ .
- (b) If the particle  $B$  reaches the table, show that the total length of the string cannot exceed  $a(1 + \sqrt{3})$ .